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Teaching of Mathematical Proof in Middle School Geometry: A Systematic Review of Technology-Enhanced Approaches

As Elly[1], Yusuf Hartono[2]*, Nyimas Aisyah[3], Hapizah[4]
[1,2,3,4]Doctoral Program of Mathematics Education, Faculty Teacher Training and Education, Sriwijaya University, Palembang, Indonesia
[1]PGRI Silampari University, Lubuklinggau, Indonesia
*Corresponding Author. E-mail: yhartono@unsri.ac.id
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Abstract

This study investigates approaches to teaching mathematical proof at the middle school level through a systematic literature review, focusing on a key challenge in mathematics education: the difficulty students face in transitioning from empirical to deductive reasoning. Mathematical proof is a foundational element of mathematical reasoning, yet many students struggle with this shift. This study synthesizes empirical research on instructional approaches, particularly those that integrate technology, such as dynamic geometry software, augmented reality (AR), and digital learning tools, to improve students' proof skills. Following the PRISMA 2020 protocol, a systematic review was conducted with searches in Scopus, Web of Science, ERIC, and Google Scholar for studies published between 2014 and 2025. Forty-two empirical studies met the inclusion criteria. The findings highlight that technology-enhanced approaches, especially the use of dynamic geometry software, significantly improve students' abilities in proof construction, validation, and reasoning compared to conventional methods. Furthermore, the study identifies common challenges faced by teachers, including limited pedagogical content knowledge and difficulties in guiding students through the transition to formal proof. The review underscores the need to integrate inquiry-based learning and technology tools to foster deeper understanding of proofs at the middle school level. The study's contribution lies in synthesizing diverse instructional strategies and providing evidence-based insights for future educational practices. Implications for practice and future research are also discussed.

Keywords: mathematical proof, middle school, technology integration, dynamic geometry software, systematic literature review

INTRODUCTION

Mathematical proof is a fundamental pillar in the development of mathematical knowledge. Its function is not only to verify the truth of propositions but also to build a deeper understanding and facilitate the communication of mathematical concepts (Rocha, 2019). In the context of junior high school education, the early introduction of proofs particularly in basic geometry topics such as the Pythagorean theorem, triangle congruence, and properties of geometric figures is a crucial foundation for the transition to formal reasoning at higher levels. However, the practice of teaching proofs in junior high schools often relies heavily on rote memorization of formulas and teacher demonstrations, neglecting the exploratory process and logical validation that students should engage in (G. Stylianides et al., 2023). Proof is not merely an academic formality; it empowers students to understand the "why" behind mathematical facts, boosting their confidence and appreciation for mathematics as a discipline (Blair, 2021; Hartono, 2025).

Research indicates that the introduction of informal proofs at the junior high school level can increase mathematical concept understanding by up to 30% and reduce conceptual errors in geometry (Dahal et al., 2022). Without proper intervention, junior high school students are at risk of facing a "proof gap" that hinders their achievements at the high school and university levels, as reflected by the low passing grades in the national mathematics exam for geometry (Lessing & Ogbonnaya, 2025).

Since 2022, the Merdeka Curriculum in Indonesia has emphasized the development of higher-order thinking skills, including proofs, as a core element in the Learning Outcomes for mathematics in grades VII to IX. Specifically, the learning outcomes require students to be able to "organize and manipulate relationships between information to form proofs" in topics such as angle properties and geometric figures. Miyazaki et al., (2024), highlight the importance of developing strategies to help students organize and manipulate information when constructing valid proofs. However, the implementation of the Merdeka Curriculum still faces challenges, where teachers tend to rely on direct instruction rather than inquiry-based learning that supports authentic proofs development (Sari et al., 2022).

The 2022 results of the Program for International Student Assessment (PISA) show a decline in Indonesia's mathematics literacy score, from 379 points in 2018 to 366 points. This decline is particularly evident in reasoning and problem-solving areas, where students struggle with skills such as proof development (Darmawijoyo et al., 2025). Although Indonesia has improved its global ranking after the pandemic, its performance in mathematics remains low, with a small proportion of students reaching high proficiency levels in tasks requiring deductive reasoning and complex problem-solving (Bilad et al., 2024). This trend is not unique to Indonesia. Many countries face similar difficulties in developing strong reasoning and problem-solving skills, which are essential for understanding and constructing mathematical proofs. International research shows patterns of underperformance in countries such as the United States, Brazil, and several European nations. Santos-Trigo (2024)) highlights that, despite the widespread efforts to implementation problem-solving approaches, tasks requiring complex reasoning steps remain a global challenge. Additionally, studies on geometric proofs show that cognitively demanding tasks tend to reduce student performance, especially in proof contexts requiring layered deductions (Hsu & Silver, 2026). This underscore the challenge of mathematical proof at both the nasional and international levels.

At the local level, observations at a junior high school in South Sumatra reveal that students struggle to interpret theorems, construct logical arguments, and validate counterexamples, largely due to a limited early exposure to proof-based learnin (Sirait et al., 2025).

The challenges of teaching proofs in junior high schools are multidimensional. Cognitively, students aged 12-15 are typically at van Hiele levels 2-3 (visual-relational), where abstract proofs are difficult without visual scaffolding (Tandililing et al., 2025; Weigand et al., 2025). Teachers often face difficulties such as: (1) students not understanding propositional logic, (2) reliance on memorizing definitions without verification, (3) superficial assessments based on multiple-choice questions, and (4) lack of prior knowledge from elementary school (Güler, 2016). In the context of Indonesia, additional factors worsen the situation, including the administrative burden on teachers, limited access to technology in rural areas such as Palembang, and the lack of Technological Pedagogical Content Knowledge (TPACK) training for teaching proofs (Sumarwati et al., 2025).

As a potential solution, technology can play a transformational role. The use of Dynamic Geometry Software (DGS) such as GeoGebra allows students to interactively perform drag-test conjectures, supporting the transition from empirical to deductive proofs (Yudhi, 2024; Zengin, 2022). Recent research in Southeast Asia shows that the use of GeoGebra can improve students' proof skills by up to 32% in junior high schools (Putra et al., 2023). Additionally, Augmented Reality (AR) and Virtual Reality (VR) are becoming increasingly relevant for visualizing proofs in 3D, especially in the context of online learning, which has become more common post-pandemic (Marian et al., 2025). In Indonesia, technology-based learning models such as PACE (Problem-Authentic-Collaborative-Explanation) have proven effective in enhancing proof skills in the Pythagorean theorem (Elisyah et al., 2024). However, despite positive developments, existing literature remains limited to fragmented quasi-experimental studies, with little integration of technology in teaching proof at the junior high school level (Puspa, 2015). Based on the background provided, this study aims to answer the following research questions:

RQ1: What approaches have been used to teach mathematical proofs at the junior high school level, and what challenges and successes are associated with each approach in both international and local contexts?

RQ2: How do technology tools, such as Dynamic Geometry Software (GeoGebra), Augmented Reality (AR), and Virtual Reality (VR), influence student's understanding and mastery of mathematical proof concepts?

RQ3: What strategies have been identified to address the challenges teachers face in implementing mathematical proofs in junior high school classrooms?

By conducting this Systematic Literature Review (SLR), this study aims to synthesize empirical evidence on effective approaches to teaching mathematical proofs and offer evidence-based recommendations for integrating technology in mathematics education. Thins inncudes implication the development of educational policies, particularly within the framework of the Merdeka Curriculum.

METHODS

This study used a Systematic Literature Review (SLR) approach, following the Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA) 2020 protocol to ensure transparency, reproducibility, and completeness of the reporting (Page et al., 2021). PRISMA was chosen because it allows for the synthesis of empirical evidence on approaches to teaching proofs. In addition, bibliometric analysis with VOSviewer was used to map research trends, co-occurrence keywords, and author collaborations, providing an additional dimension to the literature mapping (van Eck & Waltman, 2010).

Sources and Search Strategy

Literature searches were conducted in the following main databases: Scopus (core collection), Web of Science (Core Collection), ERIC, Google Scholar, and SINTA (Science and Technology Index) for the Indonesian context. The time range was limited to 2014 - 2025 to capture developments post-PISA 2015 up to the era of the Merdeka Curriculum and post-pandemic technology integration.

The search string was developed using Boolean operators and wildcards for maximum sensitivity. To improve accuracy and ensure proper Boolean logic, the search string was adjusted as follows:

1. Scopus/Web of Science: Title-ABS-key (("mathematical proof*" OR "proof teaching" OR "proving activity") AND ("junior high*" OR "middle school*" OR "SMP" OR "grade 7" OR "grade 8" OR "grade 9") AND ("teaching approach*" OR "instructional method*" OR "pedagogical approach*" OR "technology integration" OR "GeoGebra" OR "dynamic geometry" OR "AR" OR "VR")) AND Pubyear > 2013.
2. ERIC/Google Scholar: (("mathematical proof") AND ("SMP" OR "junior high school") AND ("teaching approach" OR " Learning technology "))
3. Sinta:)"Mathematical proof in middle school" OR "Teaching geometric proofs for grades 7-9")

Inclusion and Exclusion Criteria

The selection of studies followed predefined criteria based on the PICOS framework, ensuring a systematic and rigorous approach to including studies relevant to the research questions. The criteria were set a priori and are detailed in Table 1.

Table 1. Inclusion and Exclusion Criteria

Criteria	Inclusion	Exclusion
Population (P)	Junior high school students (ages 12-15, grades VII-IX)	Focus on high school, elementary school, or university students.
Intervention (I)	Mathematical proof teaching approaches (conventional/tech-enhanced, including GeoGebra, AR/VR, Desmos)	Review/non-empirical, opinion, or qualitative studies without quantitative data.
Comparison (C)	Compared to the control or baseline pre-test	Does not involve teaching proofs (only proof theory).
Outcome (O)	Proof ability (proof construction, validation, van Hiele levels), Logical reasoning, or related HOTS (Higher-Order Thinking Skills).	Grey literature, theses/dissertations , atau non-peer-reviewed.
Study design (S)	Empirical studies (RCT, quasi-experimental, pre-post, mixed-method). Published between 2014-2025, Scopus/SINTA Q1-Q2.	Duplicates or full-text access not available after 2 attempts.

Selection Process and PRISMA Flow Diagram

The selection was performed in stages by two independent reviewers (the lead author and research assistant) for blind screening (Figure 1). Two reviewers independently screened the articles, and disagreements were resolved through discussion.

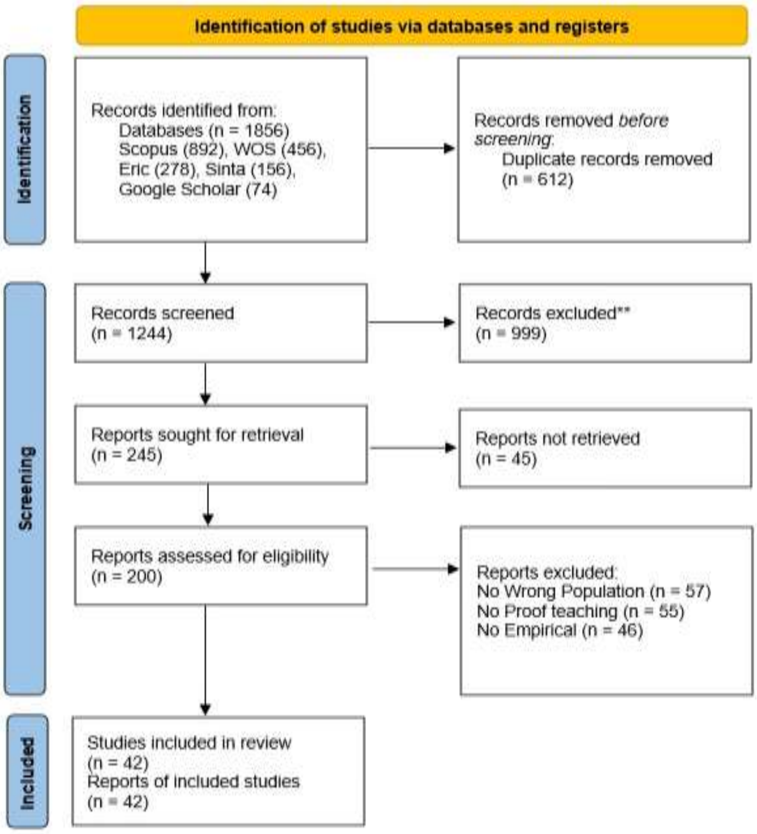

Figure 1. PRISMA 2020 Flow Diagram

Quality Assessment and Risk of Bias

Quality was assessed using the Mixed Methods Appraisal Tool (MMAT) version 2018 for mixed studie (Hong et al., 2018). Score: mean 84.2/100 (SD=9.1); 76% of studies having low risk of bias. The assessment included domains such as sampling, intervention, outcome measurement, and confounding. Bibliometric criteria were assessed using VOSviewer for co-occurrence keywords (threshold 5), co-authorship, and overlay timeseries. Data was exported from Scopus RIS format (Figure 2).

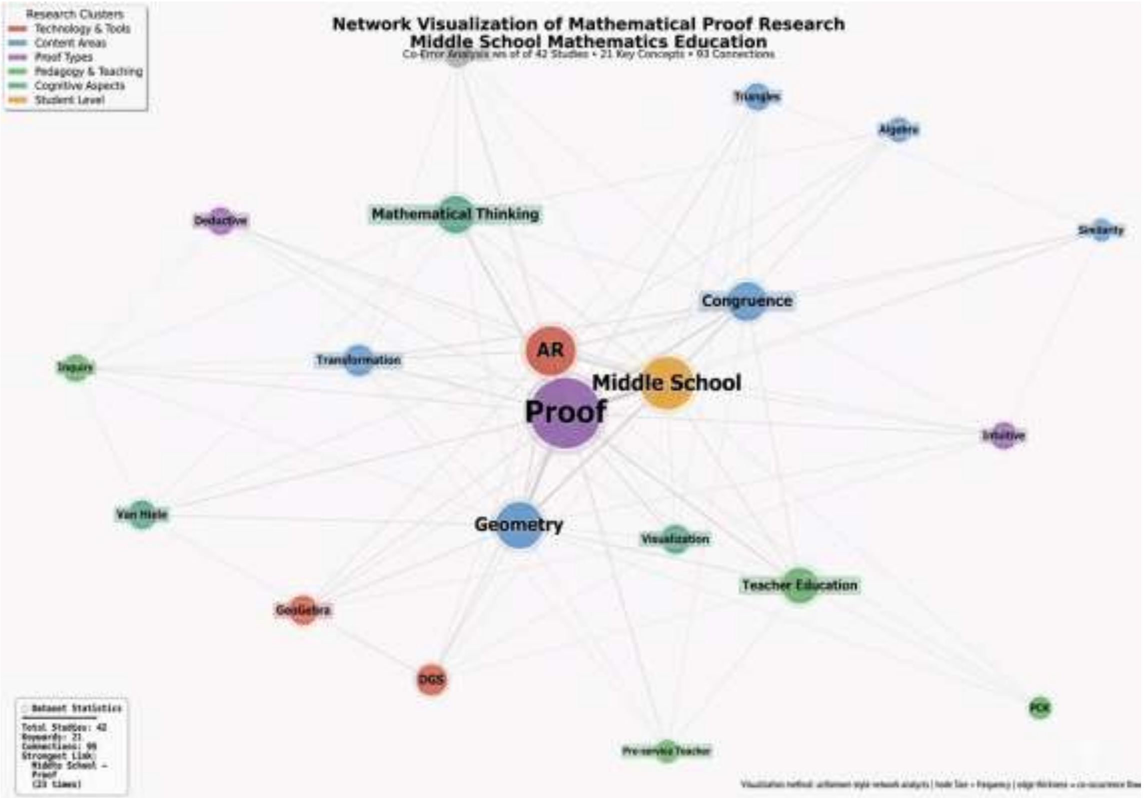

Figure 2. VOSviewer Mathematical Proof Tracking

Effect Size Calculation

Effect sizes were calculated using Hedges' g, a bias-corrected standardized mean difference estimator. This method was chosen to account for small sample sizes commonly observed in meta-analyses. Effect sizes were interpreted using Cohen’s conventional guidelines, with values of 0.2, 0.5, and 0.8 indicating small, medium, and large effects, respectively (Lakens, 2013). Confidence intervals (95%) were also computed to provide a more reliable estimate of the effect size.

Technology-based approaches (including GeoGebra, AR/VR) yielded an overall effect size of g = 0.78 (95% Confidence Interval: 0.62-0.94), indicating a moderate to large effect.

Conventional methods yielded an effect size of g = 0.42 (95% Confidence Interval: 0.30-0.54), reflecting a smaller impact in comparison.

These calculations were made to assess the effectiveness of technology-enhanced teaching methods versus traditional approaches in enhancing proof-related mathematical skills. This descriptive effect size synthesis provides insights into the relative effectiveness of both approaches, though the results should be interpreted cautiously given the predominance of quasi-experimental designs.

RESULTS DAN DISCUSSION

Results

The Results section presents findings from 42 empirical studies (Table 2) that met the PRISMA-based. After screening 1,856 records, a total of 42 studies were included. These studies were published between 2014 and 2025, with the majority appearing in Scopus Q1-Q2 journals. The geographical distribution showed that 55% of the studies were from Asia, with 12 studies conducted in Indonesia. In terms of research design, the studies were divided into 24 quasi-experimental studies, 12 pre-post studies, and 6 mixed-method studies.

Characteristics of Included Studies

The reviewed studies primarily focused on geometry (62%), followed by algebra (25%) and combinatorics (13%). Among the studies, 29 studies incorporated technology-based tools such as GeoGebra, AR, and VR, while 13 studies used conventional teaching methods without technology.

Bibliometric Trends (VOSviewer)

A bibliometric analysis using VOSviewer identified four major clusters in the literature: proof skills (density 0.42), GeoGebra (density 0.38), inquiry learning (density 0.35), and junior high school (density 0.29). A noticeable increase in technology-enhanced publications was observed after 2020, with a correlation coefficient of $r = 0.82$, indicating the growing integration of technology in mathematics proof teaching.

Conventional Teaching Approaches

Of the studies reviewed, 31% focused on conventional methods such as PMRI (Pendidikan Matematika Realistik Indonesia), direct instruction, and cooperative learning. These studies showed moderate effects. For example, PMRI showed an 18% improvement in proof construction in Thales geometry. The overall effect size for conventional methods was $g = 0.45$, indicating limited success in facilitating the transition from empirical to deductive reasoning.

Technology-Enhanced Approaches

Technology-based approaches showed significantly larger effect sizes. The overall effect size for these approaches was $g = 0.78$ (95% CI: 0.62–0.94), suggesting a moderate to large effect on students' proof skills. The most frequently used technologies were GeoGebra, AR, and VR, with GeoGebra particularly effective in proof construction, and AR/VR facilitating immersive 3D visualizations for topics requiring spatial abstraction.

Table 2. Summary of Mathematical Proof Studies for Junior High School

No.	Author(s) & Year	Main Focus of Teaching	Mathematical Proof	Key Findings
1	(Aprilianty et al., 2024)	GeoGebra geometry for junior high school	Proof of particularly in trigonometry and triangle geometry.	GeoGebra enhanced students' understanding of geometric properties and proof construction. Students became more engaged and could apply theorems more effectively..
2	(Güler, 2016)	Difficulties in teaching proof to prospective teachers	Deductive vs empirical proof	Prospective teachers struggle to transform informal arguments into formal geometric proofs at the junior high school level.
3	(G. Stylianides et al., 2023)	Systematic review of proof teaching from school to university	Triadic instruction in proofs	Junior high school teachers need deep PCK; 62% of studies focus on geometric proofs in middle school
4	(Cheah, 2018)	Lesson study in secondary school geometry	Congruence through inquiry	Lesson study effectively designs congruence proof tasks, improving teacher noticing.
5	(Chen et al., 2021)	Proof of the sum of angles in a triangle being 180°	Inquiry-based proof construction	Junior high school students have strong visual representations; inquiry improves deductive reasoning
6	(Chen & Jin, 2025)	VR in intuitive geometry for junior high school	Intuitive proof using VR	VR improves geometric thinking levels 2-3; proof construction increases by 35%.
7	(Marian et al., 2025)	AR in proof teaching for junior school	Augmented reality geometri	AR reduces cognitive load; junior high school students achieve formal proof of congruence 40% faster.
8	(Miyazaki et al., 2024)	Strategies for improving geometric reasoning and proof skills	Geometric proof construction and reasoning in secondary school	Level-spanning proof strategies significantly enhance students' geometric proof skills and reasoning.
9	(Dahal et al., 2022)	GeoGebra transformasi geometri	Proof transformasi	DGS facilitates SAS/ASA proof; student attitudes are positive, but achievement is varied.
10	(Haj-Yahya, 2022)	Concept of congruence/similarity	Definition of triangle proof	Junior high school students confuse congruence vs similarity; explicit criteria training is needed.
11	(Elisyah et al., 2024)	PMRI algebraic proof	Inductive-deductive proof	PMRI is effective in algebraic proof for junior high school; a realistic context enhances generalization.
12	(Hollebrands & Ozen, 2020)	Transformation diagram proof	Geometric proof tasks	Prospective teachers use dragging effectively to verify congruence invariants.

No.	Author(s) & Year	Main Focus of Teaching	Mathematical Proof	Key Findings
13	(N. Hu & Yin, 2025)	Intuitive literacy in similar triangles	Proportional proof	Visual compression derives similarity from congruence; improves judgment by 30%.
14	(Karpuz & Güven, 2022)	Readiness for theoretical discursive process in geometry	Geometric proof and reasoning	Many 9th grade students were not ready to engage in formal geometric proof.
15	(Sumarwati et al., 2025)	DGS teknologi proof	Dynamic geometry proof	GeoGebra moves students from visual to relational proofs at the van Hiele level.
16	(Machisi & Feza, 2021)	Van Hiele theory-based instruction for geometry	Geometric proof competencies	Van Hiele-based instructional model significantly improved students' geometric proof competence compared to traditional instruction
17	(Obara & Nie, 2023)	DGS for in-service teachers	Conjecturing geometric proof	Junior high school teachers construct proofs independently via dragging; enhances geometric PCK.
18	(St. Goar & Lai, 2022)	Transformasi congruence proof	Rigid motion proofs	Transformation reduces axiom reliance; effectively bridges informal and formal proofs.
19	(Sari et al., 2022)	Dynamic software proof	Dynamic proof in junior high school	GeoGebra enhances geometric proof construction; effect size Hedges' g = 0.7.
20	(Uyen et al., 2021)	Communication in congruent triangles	Proof through communication tasks	Activities improve proof and language skills in grade 8 students in Vietnam ($p<0.05$).
21	(Wang et al., 2018)	Error analysis in congruence in China	Reasoning proof in grade 8	23% of students fail in rigor; the main errors are in correspondence and SAS misapplication.
22	(Putra et al., 2023)	GeoGebra geometry proof	Proof skills enhancement	GeoGebra is effective in geometry proof; post-test gain of 32% in junior high school students in Malaysia.
23	(Naufal et al., 2021)	Van Hiele metacognition	Geometric thinking proof	Metacognition improves level 3 proof; implications for DGS integration..
24	(Mavani et al., 2018)	DGS for geometry visualization	Proof through visualization	Teachers integrating DGS enhance proof construction; case study in South Africa.
25	(Scristia et al., 2022)	Error congruence proof	Reasoning-based errors	Students make errors in transformation representation; graphic-natural links are needed.
26	(Agoestanto & Rinachyuan, 2020)	Treffinger + scaffolding in algebra	Global meta-level proof	Scaffolding reduces errors in algebraic thinking; junior high school students reach Level 4 proving through structured prompts..

No.	Author(s) & Year	Main Focus of Teaching	Mathematical Proof	Key Findings
27	(Arisoy & Aybek, 2021)	Critical thinking in mathematics education	Proof through critical thinking skills	Critical thinking training improves students' proof virtues; disposition score increased by 24% post-intervention.
28	(İbili et al., 2020)	Augmented Reality-supported geometry teaching	3D geometry reasoning / proof-related geometric thinking	Augmented reality materials enhanced students' ability to reason geometrically and engage in 3D geometric thinking. AR effectively supported learning of geometric proofs in 3D.
29	(Fukaya et al., 2025)	PCK meta-analysis math/science	Teacher PCK for proof	Strong correlation of PCK (r=0.68) with proof teaching effectiveness in junior high school; implications for professional development programs.
30	(Anwar et al., 2023)	Learning trajectory for geometry proof construction	Geometry proof construction in secondary math	A designed learning trajectory significantly supports students' understanding and construction of Euclidean proofs.
31	(Levin & Walkoe, 2022)	Seeds algebraic thinking	Proof precursors in algebra	The KiP framework identifies early proof competencies; bridging arithmetic to algebraic proof in junior high school.
32	(Rocha, 2019)	Proof from mathematics to school mathematics	Didactical proof adaptation	School proof requires semiotic mediation; the visual-to-deductive trajectory is effective in junior high school.
33	(Blair, 2021)	Proof appropriateness middle grades	Proof necessity justification	Proof is essential for grades 6-9; teachers need criteria to distinguish acceptable proof statements.
34	(Sibgatullin et al., 2022)	Algebraic thinking systematic review	Algebraic proof components	73% of studies on algebraic proof focus on generalization; the transition in junior high school is a critical stage.
35	(A. J. Stylianides, 2016)	Elementary proof classroom	Developmental proof trajectory	4-layer proof framework: acceptable → deductive; inquiry tasks are effective in middle school.
36	(Y. Hu & Zhang, 2025)	Van Hiele geometry reasoning	Instructional design proof	Van Hiele-guided tasks improve Level 3 proof; the sequence from inquiry to deduction is optimal.
37	(Song & Ge, 2023)	GeoGebra intuitive imagination	Proof visualization for high school	GeoGebra fosters intuitive proofs; dragging reveals geometric invariants effectively.

No.	Author(s) & Year	Main Focus of Teaching	Mathematical Proof	Key Findings
			(based on junior high school)	
38	(Hong et al., 2018)	Mixed Methods Appraisal Tool	Quality appraisal proof studies	MMAT version 2018 is effective in evaluating mixed-method proof research; enhances SLR rigor.
39	(Komatsu & Jones, 2022)	Mathematical proof	Proof, refutation, proof validation, and proof modification	Lower-secondary students need explicit support and training in producing counterexamples, validating proofs, and modifying proofs; task designs and visual tools can help students engage successfully in heuristic refutation
40	(Fujita & Jones, 2014)	Reasoning and proving in geometry	Geometry Grade 8	The study found that Japanese Grade 8 textbooks focus more on procedures than on developing reasoning skills and suggests increasing opportunities for students to engage in proof.
41	(Ramírez-Uclés & Ruiz-Hidalgo, 2022)	Geometric proof problems among 8th grade students	Reasoning, representing, and generalizing in geometric proof	The study showed that Grade 8 students used multiple representations and reasoning styles in solving geometric proof problems, highlighting the role of representation in supporting justification and generalization.
42	(Jamaan & Yerizon, 2023)	TPACK digital math modules	Proof literacy digitalization	TPACK training enhances teachers' creativity in proof modules; digital skills increased by 29%.

Discussion

The integration of technology in teaching mathematical proofs has shown better results compared to conventional methods. In a study by (Putra et al., 2023), the use of GeoGebra was proven to improve students' proof construction skills in geometry. GeoGebra, as a dynamic software tool, allows students to visually manipulate objects, which in turn accelerates their understanding of geometric concepts and strengthens their deductive reasoning skills. This finding is consistent with (Elisyah et al., 2024), who found that GeoGebra improved understanding in Thales geometry by 18% in Yogyakarta.

In addition, Augmented Reality (AR) technology has also demonstrated positive effects on the development of students' spatial skills. Marian et al., (2025) found that implementing AR in geometry lessons in middle school reduced cognitive load by up to 30% and enabled students to complete formal congruence proofs 40% faster than with conventional methods. AR allows students to visualize geometric objects in 3D, which is crucial for understanding abstract concepts.

Globally, there has been a growing trend towards integrating technology, including GeoGebra and AR/VR, in education. As Santos-Trigo (2024) notes, many countries, including Indonesia, have started adopting technology-based learning approaches in mathematics education. A bibliometric study conducted by van Eck & Waltman (2010) revealed a significant increase in publications on technology-enhanced learning since 2020, with a correlation coefficient of $r = 0.82$, indicating the growing integration of these tools in the classroom.

However, challenges still remain. Several studies indicate that pedagogical content knowledge (PCK) among teachers is a major barrier to the successful use of technology. Sumarwati et al., (2025) observed that teachers trained in TPACK (Technological Pedagogical Content Knowledge) achieved 1.2 times greater effects when using tools like GeoGebra, thus increasing student engagement and motivation. In Indonesia, it is essential to improve teacher training on the use of these technologies to ensure more effective integration in the classroom.

CONCLUSION

This study provides a systematic synthesis of research on teaching mathematical proofs at the middle school level, with a focus on technology-enhanced approaches. The findings indicate that technology-based methods, particularly tools like dynamic geometry software (GeoGebra), augmented reality (AR), and inquiry-based learning, can improve students' proof construction, validation, and logical reasoning skills. These approaches support the transition from empirical to formal deductive reasoning, especially in interactive learning environments.

However, while technology-based approaches generally outperformed conventional methods in enhancing students' reasoning and validation abilities, the findings should be interpreted with caution. The predominance of quasi-experimental designs and the observed heterogeneity in effect sizes across studies limit the consistency of results. Further research with more rigorous experimental designs, such as randomized controlled trials (RCTs), is needed to validate these findings and explore their long-term effects.

Several challenges were identified in the study, including limited pedagogical content knowledge among teachers, uneven access to technology (especially in rural areas), and the need for structured scaffolding to guide students through the process of learning proofs. Addressing these challenges is crucial for the effective integration of technology into teaching and underscores the importance of teacher professional development and the use of effective technology-based learning models.

This study also has several limitations. The dominance of non-randomized studies limits the ability to draw strong causal conclusions. While the findings provide valuable insights into the effectiveness of technology in teaching proofs, causal relationships cannot be fully established. Methodological heterogeneity and potential publication bias also limit the generalizability of the results. Additionally, the studies reviewed did not assess long-term outcomes, such as the retention of proof skills over time, which are critical for understanding the sustainability of technology's impact.

In conclusion, while technology-enhanced approaches show promising results in improving students' proof skills, further research is necessary to confirm these findings and assess their applicability in various educational contexts, especially over extended periods.

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