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Learning Design for Combinatorics with Realistic Mathematics Education (RME) Approach

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Abstract

The purpose of this research is to design learning combinatorics sub-material faktorial, permutation, combination and chance with RME approach. This research is a qualitative research. With this research method using design research. The research techniques used are measurement techniques, direct observation and documentation with tools in the form of objective tests, observation sheets and document sheets. The research subjects were 2 classes, namely A1 and A2 classes in the Mathematical Statistics course, especially combinatorics material totaling 35 students with the learning trajectory in the form of numbers that were applied alternately to students. The conclusion of this research is obtained: 1) in the faktorial sub-material, students are introduced to the RME approach that $4! = 24$, $3! = 6$, $2! = 2$, $1! = 1$ *until it is proven that* $1! = 1$*; 2) in the permutation and combination sub-material, students are directed to be examples of the same numbers and letters that form the rules of permutation and combination; 3) in the opportunity sub-material, students are exemplified about coins and dice, both in one, two, and three pieces in the context of the problem.*

***Keywords**: learning design, rme approach, design research*

INTRODUCTION

The education system in the 21st century is heading towards a revolution in producing quality human resources (HR). The influence of the industrial revolution 4.0 and preparation for the 21st century skills system specifically includes digital era literacy, creative thinking, inventive thinking, effective communication, high productivity, and spiritual norms and values (Kusumawardani et al., 2020); (Hamid & Haka, 2021). When improving the quality of education and the quality of human resources, efforts are needed from higher education institutions as the owner of authority in the world of education. As prospective teachers, students are specifically required to have excellence in knowledge and skills (Irawan et al., 2021); (Hamid & Haka, 2021).

Mathematics learning in the school curriculum is organized into several areas that classify mathematics as a compartmentalized discipline with an overemphasis on calculations and formulas. As a result of this organization, it is difficult for learners to see mathematics as a scientific field that is constantly evolving and spreading to new fields of application (Ahmad & Asmaidah, 2018); (Wittmann, 2020). Learners are positioned to see overarching concepts and relationships, so mathematics appears as a collection of fragmented pieces of factual knowledge. Today's mathematics is a growing activity around the world. It is an essential tool for many other fields such as banking, engineering, manufacturing, medicine, social sciences, and physics (Manurung et al., 2019); (Wijers & de Haan, 2020).

The goal of OECD/PISA (Lange, 2006); (Rahmawati & Mahdiansyah, 2014) to assess students' ability to solve real problems, the strategy is to define the scope of material to be assessed with a phenomenological approach to describe mathematical concepts, structures, or ideas. This means describing the content in relation to the phenomenon and type of problem being created. This approach ensures a focus in the assessment that is consistent with the domain definition, but includes a range of what is typically found in other mathematics assessments and in the national mathematics curriculum. Meanwhile, through TIMSS (Prastyo, 2020); (Novianawati & Nahadi, 2015) there are three cognitive domains that students are expected to have, namely knowing, applying, and reasoning. In the domain of knowing includes students' understanding of the concepts and procedures required by students. The applying domain includes the ability of learners to use knowledge and concepts to solve problems. The reasoning domain includes the ability of students to solve problems that are not routine and require several steps to solve.

The material in the study is combinatorics (Afifah & Dachi, 2022); (Uripno & Rosyidi, 2019) is a branch of mathematics that is useful for calculating the number of possible arrangements of objects without having to count all possible arrangements. If a set A is divided into subsets $A_1, A_2, A_3 ..., A_n$ then the sum of the elements in set A will be equal to the sum of all the elements in each subset $A_1, A_2, A_3 ..., A_n$. Indirectly based on the principle of addition every set of $A_1, A_2, A_3 ..., A_n$ does not overlap the principle of addition no longer applies and this must be solved by the principle of inclusion-exclusion. As for overlapping sets, the material presented is very closely related to the RME approach because it is often found in everyday life.

The selection of RME (Realistic Mathematics Education) is one of the right learning alternatives to solve these problems because learning does not directly present finished goods obtained by educators into the minds of students with the process of discovering this knowledge through activities or basic illustrative examples (Fitri et al., 2022). RME is an approach to mathematics education that involves learners in developing their understanding by engaging in problems set in contexts that interest them, with educators helping learners to rediscover the mathematics they encounter (Bray & Tangney, 2016). Since its emergence in the 1960s, RME has become internationally influential in mathematics curriculum and pedagogy with five characteristics specifically associated with RME (Durand-Guerrier et al., 2012): 1) the use of meaningful contexts; 2) the development of models to help move from the original context to the formal mathematical context; 3) the guided rediscovery of mathematical concepts by learners; 4) interactivity between learners and educators; 5) viewing mathematics as an interrelated subject (Fredriksen, 2020); (Ulfah & Rejeki, 2022).

The RME approach to combinatorial learning design in this study is divided into four levels in accordance with the opinion of (Wahyudi, 2016). The four levels are depicted in Figure 1.

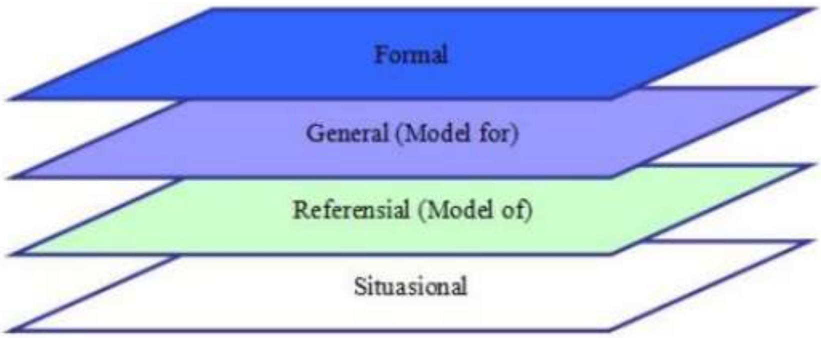

Figure 1. Four Levels in RME

The RME levels run sequentially with the following explanation: 1) The Situational Level is the foundational part which means learners apply their prior knowledge to the situation. In this study, learners were exposed to an RME context with a number sequence context. For example, it is described with 18 learners with 6 people each holding the same numbers, namely numbers 1, 2 and 3 which will be arranged into 6 arrays, namely $\{(3,2,1),(3,1,2),(2,3,1),(2,1,3),(1,2,3),(1,3,2)\}$; 2) Referential Level in the form of informal knowledge that learners have is bridged to the context of the problem. Learners create a model based on the problem context associated with their initial situation. The model is called a model of the problem situation. In this study, the level occurred in the first to third meetings carried out with each holder of the number, for example numbers 1, 2, and 3 of the 18 participants presented their respective tasks; 3) The General level is directed towards the search for mathematical solutions. The model obtained at this level is called a model for solving problems. This level aims to explain to learners that the model they made before can be used in general to solve daily problems related to combinatorial in this case specifically about Factorial, Permutation and Combination. In this meeting, the general level occurred in the fourth and fifth meetings where students used the models they had to solve other contexts, namely story problems. So, the researcher gives a story problem and learners use the model they have made before to solve (model for) the problem in the story problem; 4) The Formal Level uses mathematical symbols and representations in the mathematical concepts they build. Researchers formulate and affirm the concept of building then together with students conclude learning systematically. At this level, students are expected to have understood the definition of factorial, permutation, combination to basic probability. Then with the guidance of researchers, learners are able to develop mathematical models that resemble the basic concepts of factorial, permutation, combination, and chance.

METHODS

This research is a qualitative research type. With this research method using design research. There are two reasons, first, researchers want to apply and develop theories in mathematics learning in Indonesia in the form of local instruction theory or local instruction theory for combinatorics material that can be applied in learning with the RME approach (Wahyudi, 2016) and second because researchers want to be directly involved in understanding the way students think both in learning and solving problem problems.

Aleslami et al., (2021) explained that design research has three phases that must be carried out, namely: 1) preparation for experiments, literature studies, and hypothesizing learning trajectories; 2) teaching experiments, 3) retrospective analysis, comparing hypothesized learning trajectories and actual learning trajectories in the form of learning processes that will occur.

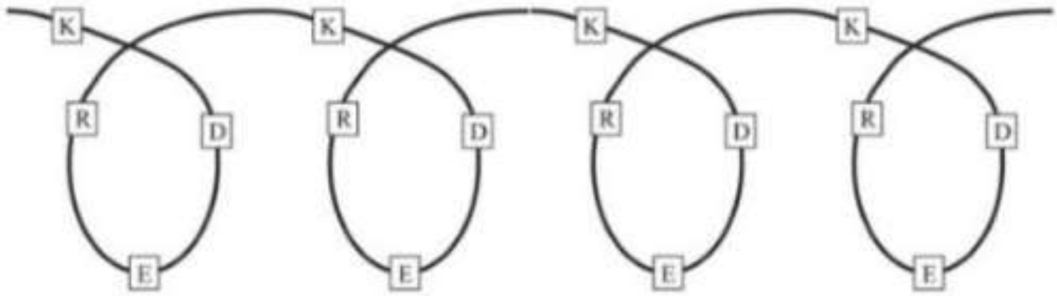

Figure 2. Design research cycle

In this study, the researcher conducted one cycle (K-D-E-R). The researcher collected knowledge about combinatorial, information and data (K), then designed a lesson for the combinatorial material (D), then conducted a teaching experiment that lasted for 4 meetings (E), then conducted a retrospective analysis of the experiments that had been carried out and the design that had been made (R), finally the results of reflection through the data that had been collected by the researcher produced a local instruction theory. The cycle aims to see the readability of the instruments that have been made, such as learning trajectory hypotheses, actual learning trajectories, research instructions and to analyse the design that has been made.

The research techniques used are measurement techniques, direct observation and documentation with tools in the form of objective tests, observation sheets and document sheets. The research subjects were 2 classes, namely classes A1 and A2 in the Mathematical Statistics course specifically combinatorics material totalling 35 students with the learning trajectory in the form of numbers that were applied alternately to students.

RESULTS AND DISCUSSION

This research was conducted in four meetings. At each meeting, the findings were documented and then described how the results of using the RME design in learning. The first meeting aimed to get students to understand combinatorial characteristics such as factorials, permutations, combinations, and probabilities. Based on the student activity sheet given, they made groupings based on the characteristics they could think of. The researcher guided the students to find combinatorial concepts by using the groupings that had been made by the students. Then, the researcher provided confirmation and information about the symbols of combinatorial writing.

In the second meeting, students were guided to discover the concepts of factorial, permutation, combination, and chance through number grouping. For the concept of factorial, students were divided into groups of numbers for each presentation of the material as exemplified in table 1.

Table 1. Arrangement of students holding each Factorial sub-material number

Factorial	Number Per Student	Structure of RME	Total
4!	The number 4 is held by 6 people The number 3 is held by 6 people Number 2 is held by 6 people Number 1 is held by 6 people	$\{(4,3,2,1)(4,3,1,2)(4,2,3,1)(4,2,1,3), (4,1,3,2), (4,1,2,3),$ $(3,4,2,1), (3,4,1,2)(3,2,4,1)(3,2,1,4), (3,1,2,4)(3,4,2,1),$ $(2,3,4,1)(2,3,1,4)(2,4,3,1)(2,4,1,3), (2,1,3,4), (2,1,4,3),$ $(1,4,3,2)(1,4,2,3)(1,3,4,2)(1,3,2,4), (1,2,4,3), (1,2,3,4)\}$	24 Ways
3!	Number 3 is held by 2 people The number 2 is held by 2 people Number 1 is held by 2 people	$\{(3,2,1), (3,1,2),$ $(2,3,1), (2,1,3),$ $(1,3,2), (1,2,3)\}$	6 Ways
2!	The number 2 is held by 2 people Number 1 is held by 2 people	$\{(2,1),$ $(1,2)\}$	2 Ways
1!	Number 1 is held by 1 person	$\{(1)\}$	1 Way
0!	Does not involve students	$\{\ \}$	1 Way

From table 1, it is agreed that there is a combined class of A1 and A2 classes totaling 35 people in one set of universes. This means that the class will involve 35 people taking turns to hold numbers and logically there is an empty set that accompanies it.

Table 2. Arrangement of students holding each number of Permutation and Combination sub-materials

Arrangement	Number Per Student	Permutation	Combination
3	Required quantity	123,132,213,231,312,321 124,142,214,241,412,421 134,143,314,341,413,431 234,243,324,342,423,432	123 124 134 234
2	Required quantity	12,21 13,31 14,41 23,32 24,42 34,43	12 13 14 23 24 34
1	Required quantity	1 2 3 4	1 2 3 4
0	Does not require students	{ }	{ }

In the third meeting, students learned about permutations and combinations. Permutation is an arrangement of a set of objects with respect to their order. While Combination is an arrangement of a set of objects without regard to their order. Their rules follow the set which is a collection of all possible objects and is certain according to predetermined rules (Nar Herrhyanto and Tuti Gantini, 2009). Students hold each number from 1 to 4 and then collect them according to the rules of permutation and combination.

Students' activities in utilizing RME learning design on the subject of permutation and combination are designed in such a way that each student plays a different role. The arrangement of student roles in learning can be seen in table 3.

Table 3. Arrangement of students holding each Opportunity sub-material number

Sample Space	Number Per Student	Chance of Occurrence
Single Coin	1 Students holding Numbers 1 Students holding Letters	$1/2$ for Numbers $1/2$ for Letters
Double Coins	4 Students holding Numbers (A) 4 Students holding Letters (H)	$1/4$ for AA (Number Number) $1/4$ for AH (Number Letter) $1/4$ for HA (Letter Number) $1/4$ for HH (Letter Letter)
Single Dice	1 Students holding Numbers 1 1 Students holding Numbers 2 1 Students holding Numbers 3 1 Students holding Numbers 4 1 Students holding Numbers 5 1 Students holding Numbers 6	$1/6$ for Number 1 $1/6$ for Number 2 $1/6$ for Number 3 $1/6$ for Number 4 $1/6$ for Number 5 $1/6$ for Number 6
Double Dices	6 Students holding Numbers 1 6 Students holding Numbers 2 6 Students holding Numbers 3 6 Students holding Numbers 4 6 Students holding Numbers 5 6 Students holding Numbers 6	$1/36$ for the arrangement $(1,1)$ . . . $1/36$ for the arrangement $(6,6)$

In the fourth meeting, students learned RME about set material. Students were given examples of coins and a dice to illustrate the situation. With students also sharing roles regarding the results of the numbers and letters into an opportunity that is exemplified. For example, S shows the sample space of the experiment and A shows the set of all events that can be formed from S.

Probability P is a function with domain A and result area $[0,1]$ that satisfies the following properties: 1) $P(A) \geq 0$, for $A \in A$, 2) $P(S) = 1$.

Furthermore, in the fifth meeting, the researcher exemplifies many permutation, combination, and chance problems into the context of the real world in everyday life, so that it is hoped that students will be able to interpret combinatorial in more detail and thoroughly. This is in line with previous research, namely: 1) (Ulfah & Rejeki, 2022) regarding the design of LKPD with the RME approach which is very feasible to use in supporting the learning process; 2) (Manurung et al., 2019) suggested that the RME approach is very suitable for use in the design of set material; 3) (Fitri et al., 2022) concluded that the RME approach was able to improve student learning outcomes.

CONCLUSION

Based on the results of research and discussion, it is obtained that the design of learning mathematics combinatorial material using the Relalistic Mathematic Education (RME) approach is by utilizing the arrangement of numbers or letters. The selection of these numbers is conditioned by the usefulness of each sub-material taught, for example, factorial, permutation, combination, and chance sub-materials with different interactions and steps used in each teaching.

The following conclusions were obtained: 1) in the factorial sub-material, students are introduced to the RME approach that $4! = 24$, $3! = 6$, $2! = 2$, $1! = 1$, until it is proven that $1! = 1$; 2) in the permutation and combination sub-material, students are directed to be examples of the same numbers and letters that form the rules of permutation and combination; 3) in the opportunity sub-material, students are exemplified about coins and dice, both in one, two, and three pieces in the context of the problem. The researcher also drew the conclusion that the RME approach is very well used and applied in learning mathematics, especially set material, logic, and combinatorial.

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