Hipotenusa : Journal of Mathematical Society
Volume 6 Issue 2 Year 2024
ISSN: 2716-3156
Website: https://ejournal.uinsalatiga.ac.id/index.php/hipotenusa/index

Representation Ability in Mathematical Modeling Based on Learning Independence

Radika Widiatmaka[1], Iqbal Kharisudin[2]*, Masrukan[3]
[1,2,3]Universitas Negeri Semarang, Indonesia
*Corresponding Author. Email: iqbalkharisudin@mail.unnes.ac.id[2]
DOI: 10.18326/hipotenusa.v6i2.2424

Article submitted: September 13, 2024
Article reviewed: November 21, 2024
Article published: December 30, 2024

Abstract

This research investigates the impact of Problem-Based Learning (PBL) on students' mathematical representation abilities within the context of mathematical modeling, with a particular focus on the role of learning independence. The research employs a mixed-method sequential explanatory model to analyze the effectiveness of Problem-Based Learning and to categorize students' abilities and independence levels. Conducted with class XI TL 2 at SMAN 1 Mranggen during the 2023-2024 academic year, the findings reveal that PBL significantly enhances students' mathematical representation abilities. Moreover, learning independence is identified in two distinct categories–high and medium–which correspond with similar levels of representation ability. Students' mathematical representation ability has two categories, namely high and medium. These insights underscore the importance of self-directed learning in fostering mathematical comprehension and skills.

Keywords: mathematical representation, mathematical modeling, learning independence, problem-based learning

INTRODUCTION

The study addresses a gap in exploring the relationship between students' representation skills in mathematical modeling and their level of learning independence. While previous research has examined these aspects separately, their direct interaction remains underexplored, particularly regarding how independence influences students' choice and use of mathematical representations. Moreover, the development of learning models and assessment methods that enhance both representation skills and independence requires further investigation. This study aims to fill this gap by examining the interplay between these factors and their impact on learning mathematical modeling, contributing to more effective mathematics education strategies.

The National Council of Teachers Mathematical stipulates that students must learn the following five process abilities through their study of mathematics: (1) problem-solving; (2) reasoning and proof; (3) connection; (4) communication; and (5)

Hipotenusa is licensed under a Creative Commons Attribution-ShareAlike 4.0 International License.

representation (NCTM, 2000). An important part of learning mathematics is learning to use language, rules, and mathematical representations. In learning mathematics, mathematical representation ability is one of the standards that must be achieved by students. According to G. Goldin representation is a configuration that has multiple methods to represent another entity. For instance, a word may stand in for a physical thing, a number representing an individual's weight, alternatively the same number may represent a position on a number line (Goldin, 2002). Goldin and Steinghold (2001) distinguished two facets of representations, namely internal and external representation (Abdul Aziz & Kurniasih, 2019; Goldin, 2002). Internal representation is the process of thinking about mathematical ideas that occur in the individual mind of a person who works based on these ideas. In general, concepts that occur in a person's mind cannot be observed with the senses. External representations are forms of thought that can be revealed either orally, in writing, through symbols, expressions, images, graphs, diagrams, tables, or through physical objects in the form of props. The teacher must promote the usage of multiple representations in the classroom in order to help the students develop their translation abilities (Nurrahmawati et al., 2021). Villegas suggests 3 types of external representations, namely: (1) Verbal representation of the problem: basically, consists of a story problem that is stated, either in writing or orally. (2) Pictorial/visual representation: consists of pictures, diagrams, or graphs, as well as related types of actions. (3) Symbolic representation: consists of numbers, operations, relation signs, algebraic symbols, and any type of action that refers to these symbols (Villegas et al., 2009). The indicators modified in this study are contained in Table 1.

Table 1. Indicators of Students' Mathematical Representation Ability

No.	Type	Indicator
1.	Visual Representation	1. Students create pictorial representations, graphs, tables, and so on, or modify previously created pictorial representations.
		2. Students find solutions with pictorial representations.
2.	Symbolic Representation	1. Students solve or attempt to solve symbolic expressions with writing.
		2. Students modify, rewrite, or eliminate symbolic expressions.
3.	Verbal Representation	1. Students diagnose the problem.
		2. Students write down steps, descriptions, or information to solve the problem.
		3. Students summarize with sentences.

The independent curriculum is an educational strategy, which strives to offer students with independence in studying. In an independent curriculum, students are expected to independently develop their potential to the fullest. It is expected that with high learning independence, students will be more creative, innovative, think critically, collaborate, and have an understanding of the latest technology. Students learning independence must be familiarized so that it becomes a culture in a person because independence is included in character education. Independence may be seen as an individual's evaluation of his capacity to carry out a sequence of actions in order to obtain the desired outcomes (Bandura, 1997). Learning independence was characterized by Bandura as a characteristic that influences a person's assessment of themselves and how their behavior manifests, specifically in relation to their ability to plan and execute the necessary actions to carry out a particular performance successfully (Cadapan et al., 2022). Learner learning independence is an implementation of a reflection of the learning attitude of students when participating in a series of learning activities in class and outside the classroom (Sari, 2024).

Mathematical modeling is the process of presenting or depicting a real-world problem in a mathematical problem in an attempt to find a solution to the problem or improve a better understanding of the problem (Ang, 2018). Mathematical modeling is a cognitively demanding activity since multiple competencies are required, including non-mathematical ones, extra-mathematical knowledge is needed, mathematical knowledge is needed, especially for translations, conceptual ideas are needed, and appropriate beliefs and attitude are needed, especially for more complex modeling activities (Blum, 2015). Through mathematical modeling strategys, students get a deeper understanding of the connection between mathematics and real-world issues (Blum, 2002; Riyanto et al., 2019). Buchholtz (2021) particularly puts emphasis on contextualized mathematization and contextualized validation. He contends that both the validation procedure and the stages of organizing and mathematizing take place within the context of the actual thing. Selecting a mathematical model might encourage independent mathematics based on real-world issues within a restricted theme framework (Buchholtz, 2021; Jablonski, 2024).

Mathematical modeling involves representing real-world problems as mathematical problems to find solutions or enhance understanding (Ang, 2018). This process is cognitively demanding, requiring various competencies, including non-mathematical ones. It necessitates extra-mathematical knowledge, mathematical understanding, conceptual ideas for translations, and appropriate beliefs and attitudes, especially for complex modeling tasks (Blum, 2015). Mathematical modeling methods help learners better grasp the connection between real-world issues and mathematics (Blum, 2002; Riyanto et al., 2019). Buchholtz (2021) emphasizes contextualized mathematization and validation, noting that structuring and mathematizing, as well as the validation process, occur within the context of the real object. Choosing a mathematical model can encourage autonomous mathematics based on real-world problems within a specific thematic context (Buchholtz, 2021; Jablonski, 2024).

Usually, modeling exercises based on a real-world event or item are introduced in the classroom. A literary introduction can be made verbally, through texts or newspaper articles (Eames et al., 2018). The mathematical modeling cycle, as described in Catalyzing Change (NCTM, 2018), begins with a real problem and involves multiple steps and often multiple iterations: 1) Formulating the problem or questions. 2) Stating assumptions (often requiring simplifications of the real situation) and defining variables. 3) Restating the problem or question mathematically. 4) Solving the problem in the mathematical model. 5) Analyzing and assessing the solution and the mathematical model. 6) Refining the model, going back to the first steps if necessary. 7) Reporting the results (NCTM, 2018, 2020). According to Blum and Leiß (2007), there are steps to learning mathematical modeling, namely: 1) Understanding: students understand the problem, problem, or picture that has been given. 2) Simplifying or structuring: students choose data or what has been given to be simplified, and arranged towards a real-world model situation. 3) Mathematization process (Mathematising), mathematization process from real models into mathematical concepts. 4) Working Mathematically, using mathematical concepts/operations to find results. 5) In interpreting, mathematical results in real language according to the given problem. 6) Validating and checking the interpreted results in the real world according to the problem; if the solution is not correct or does not make sense, it can repeat from stage 2 and so on (Ahsan et al., 2023; Blum & Leiß, 2007; Greefrath et al., 2018; Sanjari & Manouchehri, 2024; Serpe & Frassia, 2020).

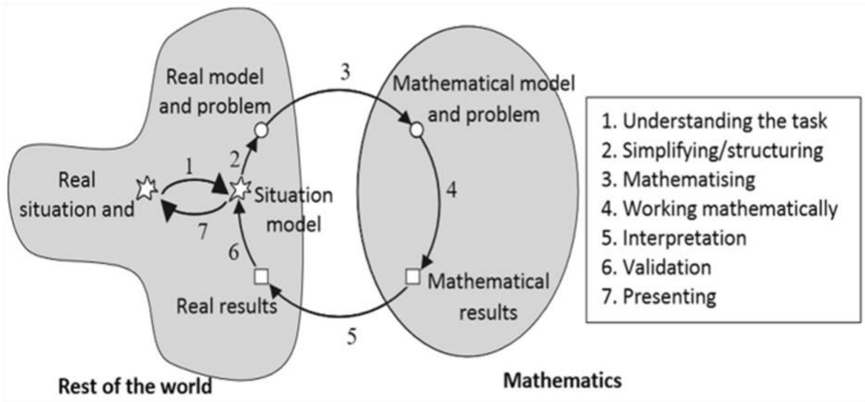


Figure 1. The Modeling Cycle of Blum and Leiß (2007)

Students build models of real-world situations to begin resolving real-world issues. Following that, students convert this model into a mathematical model and go from the physical world to the mathematical realm. After that, calculations may be performed in the field of mathematics, and the outcomes must be reviewed and verified in relation to reality. Images can aid in filling in the gaps in knowledge and highlight the connection between the issue and the outside world (Hartmann & Schukajlow, 2021). Students will develop the ability to employ a variety of mathematical representations as well as apply suitable mathematical techniques and methodologies to solve issues within the actual world through mathematical modeling. In order to describe real-world situations in a mathematical manner or create mathematical models, mathematical modeling necessitates the selection and use of suitable mathematical ideas or representations (Zulkarnaen, 2020). General steps taken to show mathematical representations in mathematical modeling according to Kharisudin & Cahyati, (2020), namely: (1) Identifying all quantities involved in real problems. (2) The identified quantities are given symbols, their units are determined (in a system of units), and the variables and constants are determined. (3) Establishes the laws that govern the issue. These rules establish a mathematical framework that explains how each variable and the constant relate to one another (4) Determines the solution of the model. (5) Interpret the model solution as if it were a problem solution (Rahman & Kharisudin, 2019). We can see the mathematical modeling scheme as presented in Figure 2 (See eg. Rahman & Kharisudin, 2019; Sutrisno & Kharisudin, 2020).

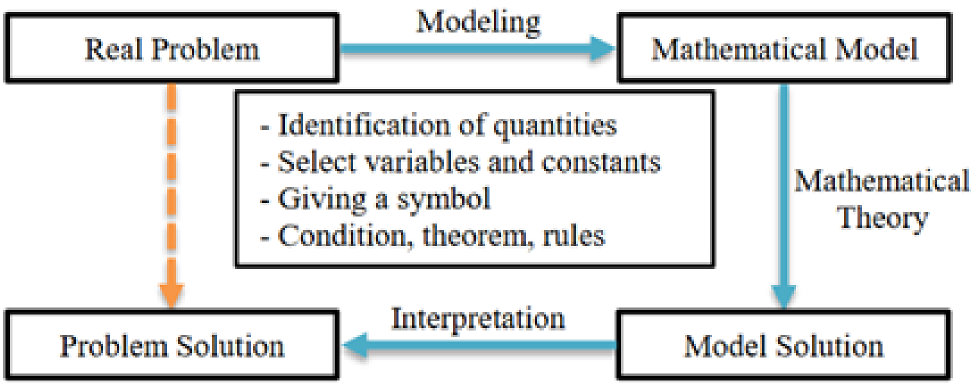


Figure 2. Mathematical Modeling Process

Learning about mathematical modeling is very strategic for developing students' problem-solving abilities (Kharisudin & Cahyati, 2020; Nurochmah & Kharisudin, 2023). Mathematical modeling is a powerful and versatile strategy for problem-solving. Many mathematical problems can be effectively addressed using a modeling approach. It is essential to introduce students to modeling, beginning with simple concepts and gradually advancing to more complex problems, tailored to their level of understanding. The modeling process begins with a thorough comprehension of the real-world or mathematical problem, It is then changed into a mathematical model through several stages. These stages include identifying quantities, selecting variables and constants, assigning symbols, and determining the governing laws. The next step involves solving the model by obtaining the values of the variables through mathematical procedures. Finally, the problem's solution is reached by finding and interpreting the answer.

In order to enhance students' knowledge and problem-solving abilities, Problem-based learning is a learner-centered teaching approach that serves as an appropriate learning environment in the real world, backed by transdisciplinary learning. It also offers solutions to unstructured problem difficulties (Falloon et al., 2022; Su, 2024). The model to learning that is problem-based learning may help students discover mathematical principles, expand their knowledge, and engage in higher level thinking. Students will be able to use critical and logical thinking to solve issues making use of the problem-based learning model (Hasanah et al., 2021; Maslihah et al., 2021). Problem-based learning is also a learning style where students work on problems to compile their knowledge, developing thinking skills and problem-solving skills, and developing independence and self-confidence. The learning syntax with the Problem-based learning model includes: a) Orienting students to the problem, meaning that the teacher describes the objectives of learning; b) proposing problems, and encouraging students to participate in problem-solving activities. b) Organizing students to learn, meaning that students are supported by the teacher in describing and grouping tasks related to the problem. c) Guiding individual and group investigations means that students are encouraged to seek information and practice as a problem-solving process. d) Developing and presenting work means that students are guided by the teacher to prepare work. e) Analyzing and evaluating the problem-solving process means that the teacher assists in carrying out reflection or assessment activities on the results of the work made (Al-Tabany, 2017).

METHODS

This research uses a mixed method, namely the sequential explanatory model. The sequential explanatory combination method is a research method that sequentially combines quantitative and qualitative methods, with quantitative methods coming first and qualitative methods following. In the quantitative method employs a quasi-experiment with a nonequivalent control group design, in which researchers utilize two groups that are not the same (nonequivalent) for each learning, namely experimental and control classes. A qualitative method was used to describe students' mathematical representation ability in mathematical modeling for learning independence. The research subjects were 36 students of class XI TL 2 SMA Negeri 1 Mranggen in the academic year 2023-2024. Data for this study was gathered through exams, questionnaires, interviews, and observations. Data on students' learning independence was gained through questionnaires, while data on students' mathematical representation skills was gathered through exams. Data was analyzed using data reduction, data display, triangulation, data interpretation, and data summarization. Credibility, transferability, dependability, and confirmability tests are all examples of qualitative data validity assessments (Creswell & Poth, 2018). The credibility test uses triangulation methods, whereas the transferability exam describes students' mathematical representation skills in Problem-based learning in depth and methodically. The dependability test is conducted by collecting student data in accordance with their learning independence and mathematical representation ability patterns, and the confirmability test is carried out by connecting the research data with existing theories and confirming the research results to experts.

To gather information on how well students can describe representation mathematically based on their independence in learning mathematical modeling, researchers set up three instruments consisting of a learning independence questionnaire, a mathematical representation ability test, and an interview. In order to determine students' levels of learning independence, a Google Forms questionnaire was created, and students were sent the link via the class WhatsApp group. Students' mathematical representation abilities were examined utilizing the mathematical representation indicators, and the results were reported in the form of a written exam description called the mathematical representation ability test. To validate the results indicating a relationship between students' learning independence and their ability for mathematical representation, researchers and study subjects performed private interviews.

RESULTS AND DISCUSSION

Learning Effectiveness on Mathematical Representation Ability

Learning effectiveness is divided into three stages according to the learning framework (Danielson, 2013), specifically, the following: (1) organizing and preparing; (2) carrying out the learning process (instruction and classroom setting); and (3) assessing the learning process (professional responsibilities). Learning is carried out using the Problem-based learning model with worksheets and the completion of mathematical modeling strategies.

Based on the results obtained, the educational resources and research instruments related to mathematical representation skills in mathematical modeling are in a good category. So it is concluded that the learning tools and research instruments are valid. In the learning implementation stage, both the average final learning implementation score and the average learning implementation score at each meeting fall into the good category. Based on these findings, it is possible to conclude that implementing problem-based learning can seek exploration of mathematical representation skills in mathematical modeling. Considering the outcomes of this effectiveness test, it was obtained (a) the average mathematical representation ability in mathematical modeling in Problem-based learning reached the minimum learning completeness of 70. With the results of the right-party average completeness test, the values of t-value = 7.937 while t-table = 1.689 at a 5% threshold of significance. (b) In problem-based learning, more than 85% of the proportion students meet the minimal learning completion requirements. Based on the z-test results obtained, z-value = 0.9145 and z-table = 0.1736, with z-value > z-table. (c) The average proficiency in mathematical representation during problem-based learning through mathematical modeling surpasses the average proficiency in mathematical representation during conventional learning. The test results obtained were t-value = 1.686 and t-table = 1.671, at a 5% threshold of significance. And (d) The experimental class with a medium category and the control class with a low category show a significant difference in efficacy when it comes to helping students improve their mathematical representation abilities. According to the results of the difference test between the two average gain scores in Table 2, it is obtained that t-value = 3.656 and t-table = 1.689 at a 5% threshold of significance, and with the gain test, the data is obtained as shown in the following table.

Table 2. Gain Score Results

Class	Gain Score			Gain Criteria
	Minimum	Maximum	Average	
Experiment	0,17	0,54	0,3314	Medium
Control	0,00	0,46	0,2442	Low

Thus, it can be concluded that Problem-based learning is effective in improving students' mathematical representation abilities in mathematical modeling.

Description of Mathematical Representation Ability Based on Learning Independence

The learning independence questionnaire is prepared based on indicators of learning independence, which include: (1) intrinsic learning initiative and motivation; (2) the habit of being able to diagnose learning needs; (3) having the ability to create learning objectives; (4) monitoring, regulating, and controlling learning; (5) perceiving challenges as opportunities for growth; (6) using and locating pertinent sources; (7) selecting and implementing learning tactics; (8) assessing learning outcomes and processes; and (9) being able to develop the concept of self-ability (Hendriana et al., 2018). Student learning independence is categorized into high, medium, and low categories (Hidayah et al., 2019).

The link to the learning independence questionnaire was distributed through the experimental class WhatsApp group and completed by all students. The findings of the students' independent learning were examined by the researcher. Students' learning independence is recognized to be in the high and medium categories based on the study findings. There are 25 students with medium learning independence and 11 with high learning independence.

Table 3. Experimental Class Student Learning Independence Category

Score Interval	Category	Total	%
25-58	High	11	30,56
59-92	Medium	25	69,44
93-125	Low	0	0

Students with high learning independence were 11 students, and students with medium learning independence were 25 students. Among the 11 students with high learning independence, nine have high mathematical representation skills, and two have medium mathematical representation skills. Among 25 students with medium learning independence, 18 students have high mathematical representation skills, and 7 students have medium mathematical representation skills.

Furthermore, Problem-based learning and the mathematical representation ability test (MRAT) were provided to the research participants after the learning of instruction. Data on students' proficiency with mathematical representation were acquired based on the final of MRAT findings, namely the high mathematical representation abilities of 27 students and the medium mathematical representation abilities of 9 students. Students' mathematical representation ability was confirmed with students' learning independence through analyzing the final results of MRAT and expanded through interviews. The researcher interviewed a number of subjects.

The following is a description of the pattern of mathematical representation ability of students with high learning independence

The following are the results of the work of student subjects with high learning independence on visual representation.

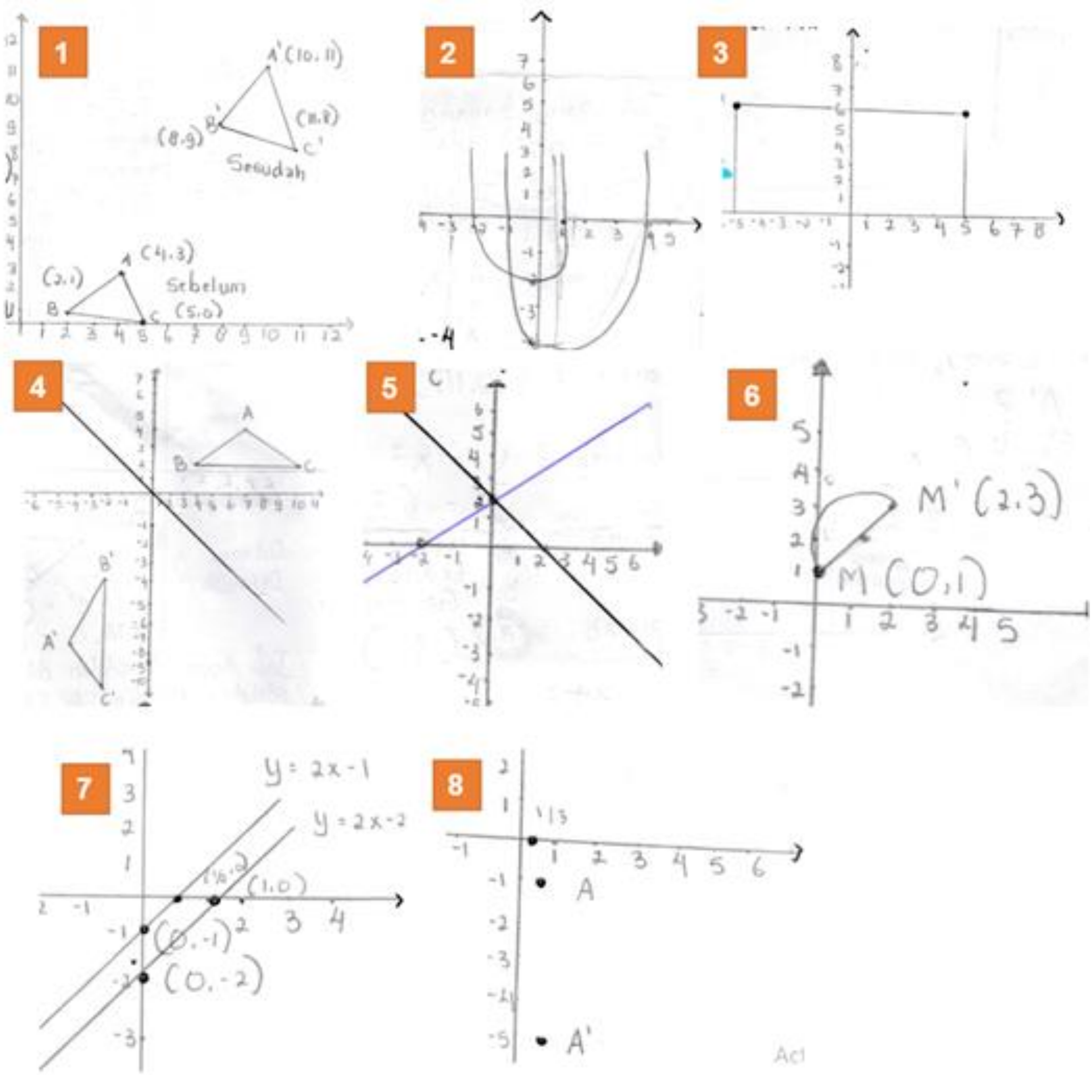


Figure 3. The result of EX-34 Subject on Visual Representation

Considering the outcomes of EX-34's written work, it appears that it has fulfilled the visual representation indicator from question number 1 to number 8 in full and is depicted graphically.

The following is the work of EX-34 subjects on symbolic representation.

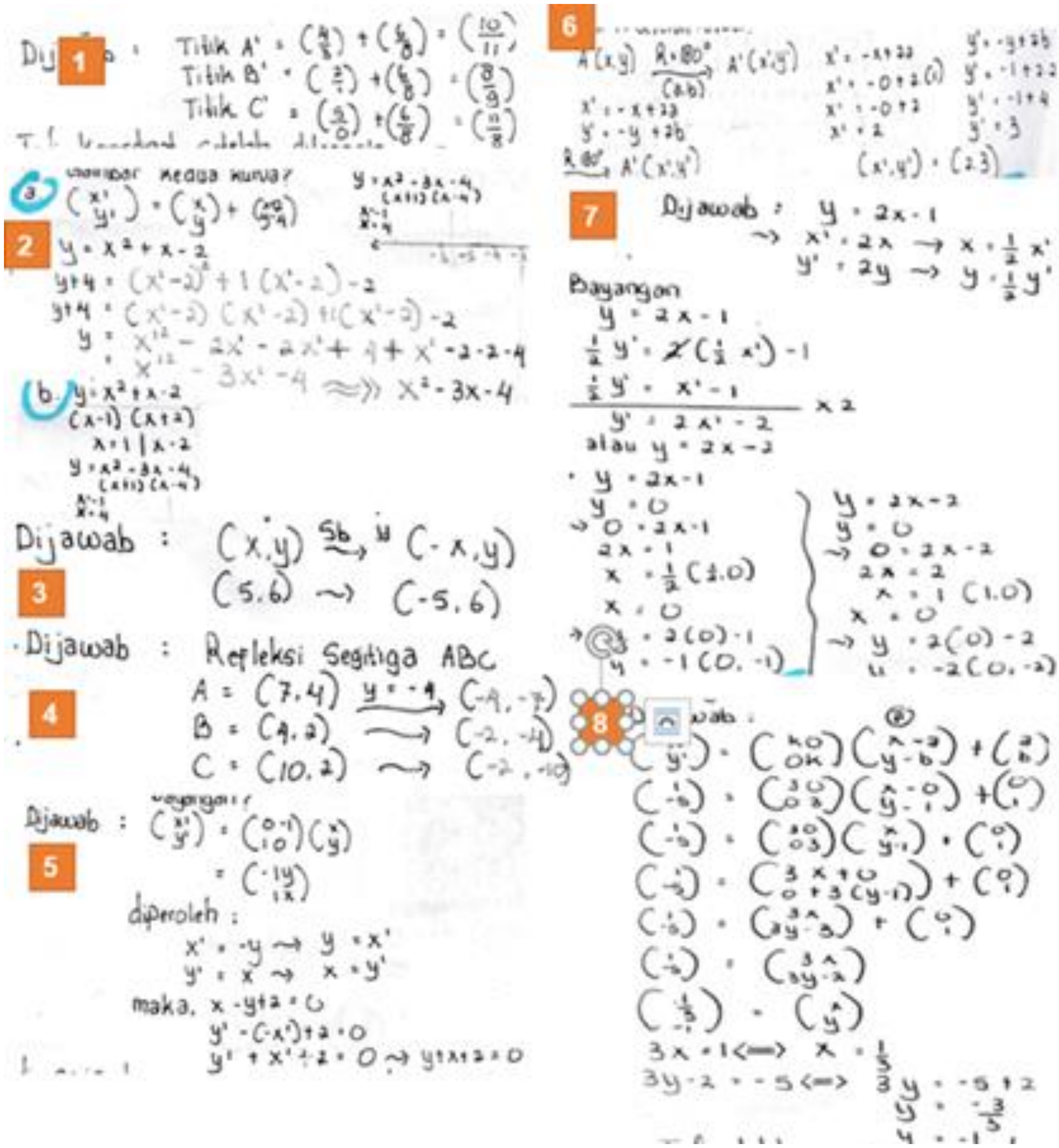


Figure 4. The result of EX-34 Subject on Symbolic Representation

Based on the results of EX-34's written work, it appears that it has fulfilled the symbolic representation indicator from question number 1 to number 8 in mathematical symbolism, in matrix, algebra, or both.

The following is the work of EX-34 subjects on verbal representation.

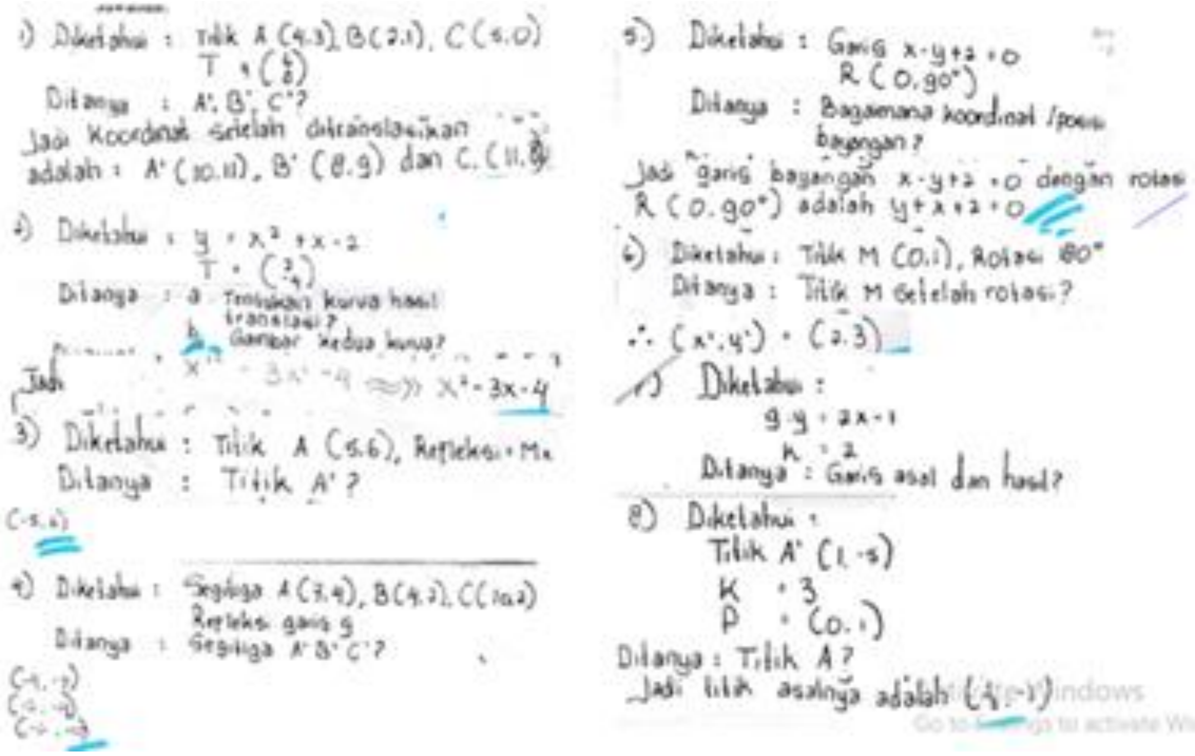
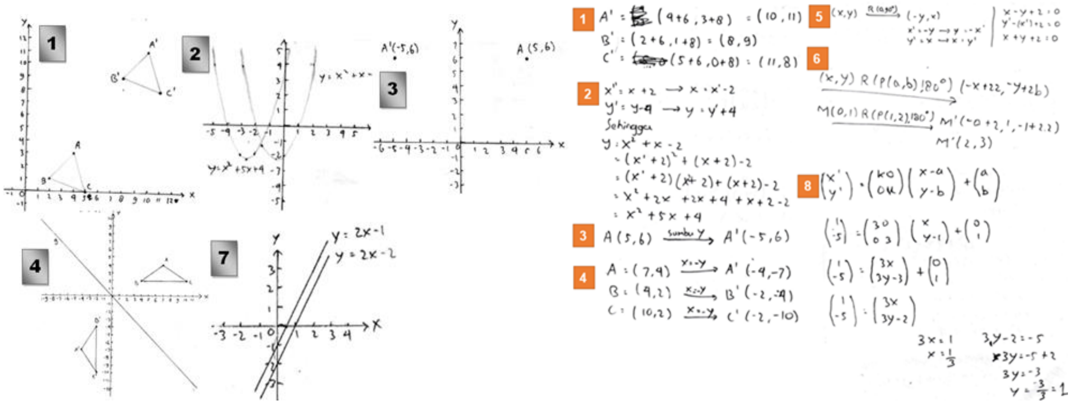


Figure 5. The result of EX-34 Subject on Verbal Representation

Based on the results of EX-34's written work, it appears that it has fulfilled the verbal representation indicator from question number 1 to number 8 can write known, asked, and conclusion.

The following is a description of the pattern of mathematical representation ability of students with medium learning independence

The following is the results of EX-21 subjects with medium learning independence on visual representation and symbolic representation.



(a) (b)

Figure 6. (a) Visual Representation of EX-21 Subject; (b) Symbolic Representation of EX-21 Subject

Based on the results the visual representation indicator from question number 1, 2, 3, 4, and 7 is depicted graphically. Based on the results of EX-21's the symbolic representation indicator from question number 1 to number 8 in mathematical symbolic, in matrix, algebra, or both directly and simply to get problem-solving. Subject EX-21's verbal representation was not visible.

Discussion

In the mathematical representation ability, many studies have analyzed representation skills as a learning activity (Bach et al., 2024; Goldin, 2002; Magana et al., 2024; Nurrahmawati et al., 2021; Scheiner & Montes, 2024; Villegas et al., 2009). The results of MRAT, Ex-34 and Ex-32 can answer nine questions correctly out of 8 available items. All questions answered fulfill all steps taken to show mathematical representations in mathematical modeling strategy (Rahman & Kharisudin, 2019), which is (1) Identifying all quantities involved in real problems. (2) The identified quantities are given symbols, their units are determined (in a system of units), and the variables and constants are determined. (3) Establishes the laws that govern the issue. These rules establish a mathematical framework that explains how each variable and the constant relate to one another (4) Determines the solution of the model. (5) Interpret the model solution as if it were a problem solution. On the other hand, based on the results about MRAT, Ex-02 and Ex-21 have not fulfilled all the steps taken to show mathematical representation in mathematical modeling strategies, namely at point (5) The model solution is interpreted as a problem solution.

The pattern of students' mathematical representation ability is described based on their multiple representations. According to previous research, multiple representations are very important for students in learning (Listiawati et al., 2023; Nurrahmawati et al., 2021; Rodriguez-Martinez et al., 2023; Villegas et al., 2009). High representation ability if it fulfills three types of representation; medium representation ability if it fulfills two types of representation; and low representation ability if it only fulfills one type of representation. The following describes the trend of students' mathematical representation skills depending on their independence in learning.

Students with high learning independence were 11 students, and students with medium learning independence were 25 students. Among the 11 students with high learning independence, nine have high mathematical representation skills, and two have medium mathematical representation skills. Among 25 students with medium learning independence, 18 students have high mathematical representation skills, and 7 students have medium mathematical representation skills.

The pattern of mathematical representation ability of students with high learning independence

Students with high learning independence were 11 students, with nine students having a high mathematical representation ability and two having a medium representation ability. The average representation ability of students with high learning independence is 83.00. Based on the interviews, students with high levels of learning independence and representation ability were discovered can fully bring out their mathematical representation ability. This condition can occur because students can utilize learning resources well. After all, they do not only learn during the implementation of learning but also deepen their abilities through practicing mathematical representation problems that are similar to examples or varied. However, some students with high learning independence have medium mathematical representation skills, that is similar to Sutrisno and Kharisudin's research, there appear to be indications of problem-solving skills and unfulfilled mathematical modeling steps even in topics with high Mathematics Self-Efficacy categories (Kharisudin & Cahyati, 2020; Sutrisno & Kharisudin, 2020).

Based on the results of EX-34, question number 2 is the most difficult about the translation of the curve, but remembering the exercises that have been done and trying to get these results. It can be concluded that EX-34 fulfills the indicators of mathematical representation virtually by drawing graphs appropriately and working with independence. Based on sample subjects, students with high learning independence, namely EX-34 and EX-2, have different representation abilities, where the EX-34 subject fulfills the visual, symbolic, and verbal representation indicators. while EX-2 subject only fulfills the visual and symbolic representation indicators. Based on the EX-2 subject interview, because he was in a hurry and not used to it, he missed the verbal representation that should be the focus study.

The pattern of mathematical representation ability of students with medium learning independence

There are 25 students who have medium learning independence, with eighteen students having a high mathematical representation ability and seven having a medium representation ability. The average representation ability of students with high learning independence is 80.00. It is concluded from the interview's findings that students with a medium-category learning independence are able to completely develop their abilities in mathematical representation. This state may arise as a result of taking learning seriously and developing one's skills by working through the provided mathematical representation challenges. Yet, there are students who exhibit a medium level of learning independence and also possess a medium level of mathematical representation abilities. Due to a missing interpretation of the model answer as a problem solution.

Considering the interview's outcomes, according to the subject EX-21, some problems were discussed with friends, and passive in practice problems and group assignments.

Considering the evaluation of the written and interview data, it is possible to conclude that EX-21 also fulfills the indicators of mathematical representation virtually by drawing graphs but not all of them work independently. Based on sample subjects, students with medium learning independence, namely EX-21 and EX-32, have different representation abilities, where subject EX-21 fulfills visual and symbolic representation indicators, while subject EX-32 only fulfills visual, symbolic, and verbal representation indicators. According to subject EX-32's interview, during the implementation of learning, he paid close attention, and deepened his abilities by practicing mathematical representation problems given when class learning was over. Table 4 presents a diagram that illustrates patterns of mathematical representation. It draws attention to the possibility that students with comparable degrees of learning independence might not all have the same kinds of mathematical representation skills.

Table 4. Pattern of Mathematical Representation Ability

Mathematical Representation Ability	Learning Independence	Pattern of Mathematical Representation Ability	Number of Students	Sample Student Code
High	High	Mastering 3 categories of representation skills (Visual, Symbolic, and Verbal)	9	EX-34
High	Medium	Mastering three categories of representation skills (Visual, Symbolic, and Verbal)	18	EX-02
Medium	High	Mastering two categories of representation ability (Visual and Symbolic)	2	EX-32
Medium	Medium	Mastering two categories of representation ability (Visual and Symbolic)	7	EX-21

CONCLUSION

Based on the research, problem-based learning has proven effective in enhancing students' mathematical representation abilities in mathematical modeling. This is indicated by the criteria: (1) The validity of the learning device validation results was determined throughout the planning and preparation phase; (2) During the learning implementation phase, the findings from the observations of the process produced excellent standards; (3) At the learning evaluation stage obtained, a) for mathematical modeling in problem-based learning, the average student's capacity to represent mathematical ideas reached the minimal level of learning completion, b) in problem-based learning, more than 85% of the proportion students meet the minimal learning completion requirements, and c) in problem-based learning, students' average proficiency in mathematical representation was superior to that of students' average proficiency in conventional learning.

The results obtained in this study are the pattern of students' mathematical representation ability in mathematical modeling in class XI SMA Negeri 1 Mranggen in terms of students' learning independence in Problem-based learning varies. The description of the pattern of mathematical representation ability is as follows: (1) students with high category learning independence have high and medium mathematical representation skills. Students with high independence and high representation ability can master virtual, symbolic, and verbal representations as well, while students with high independence and medium representation ability are able to master virtual and symbolic representations, however less mastering verbal representations; (2) students with medium learning independence have high and medium mathematical representation skills. Students with medium independence and high representation ability can master virtual, symbolic, and verbal representations well. Students with high independence and medium representation ability can master virtual, and symbolic representations, but not verbal representations.

REFERENCES

Abdul Aziz, T., & Kurniasih, M. (2019). External representation flexibility of domain and range of function. *Journal on Mathematics Education*, *10*, 143–156. https://doi.org/10.22342/jme.10.1.5257.143-156

Ahsan, M. G. K., Cahyono, A. N., & Kharisudin, I. (2023). Learning mathematical modelling with digital tools: A systematic literature review. *AIP Conference Proceedings*, *2614*(1), 040082. https://doi.org/10.1063/5.0126587

Al-Tabany, T. I. B. (2017). *Mendesain Model Pembelajaran Inovatif, Progresif , dan Kontekstual: Konsep, Landasan, dan Implementasinya pada Kurikulum 2013 (Kurikulum Tematik Integratif/KTI)*. Jakarta: Prenadamedia Group.

Ang, K. C. (2018). *Mathematical Modelling for Teachers: Resources, Pedagogy and Practice*. Routledge. https://doi.org/10.4324/9781351247979

Bach, C. C., Bergqvist, E., & Jankvist, U. T. (2024). Students' dynamic communication while transforming mathematical representations in a dynamic geometry environment. *ZDM – Mathematics Education*. https://doi.org/10.1007/s11858-024-01575-x

Bandura, A. (1997). *Self-efficacy: The exercise of control*. New York: W. H. Freeman and Company.

Blum, W. (2002). ICMI Study 14: Applications and modelling in mathematics education – Discussion document. *Educational Studies in Mathematics*, *51*(1), 149–171. https://doi.org/10.1023/A:1022435827400

Blum, W. (2015). Quality Teaching of Mathematical Modelling: What Do We Know, What Can We Do? In S. J. Cho (Ed.), *The Proceedings of the 12th International Congress on Mathematical Education* (pp. 73–96). Springer International Publishing. https://doi.org/10.1007/978-3-319-12688-3_9

Blum, W., & Leiß, D. (2007). How do Students and Teachers Deal with Modelling Problems? In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical Modelling* (pp. 222–231). Woodhead Publishing. https://doi.org/10.1533/9780857099419.5.221

Buchholtz, N. (2021). *Students' modelling processes when working with math trails—Processos de modelação de alunos envolvidos em trilhos matemáticos*. *30*, 140–157. https://doi.org/10.48489/quadrante.23699

Cadapan, R. R., Tindowen, D. J., Mendezabal, M. J., & Quilang, P. (2022). Graduate school students' self-efficacy toward online learning in the midst of the COVID-19 pandemic. *International Journal of Evaluation and Research in Education (IJERE)*, *11*(2), Article 2. https://doi.org/10.11591/ijere.v11i2.21856

Eames, C., Brady, C., Jung, H., Glancy, A., & Lesh, R. (2018). Designing Powerful Environments to Examine and Support Teacher Competencies for Models and Modelling. In R. Borromeo Ferri & W. Blum (Eds.), *Lehrerkompetenzen zum Unterrichten mathematischer Modellierung: Konzepte und Transfer* (pp. 237–266). Springer Fachmedien. https://doi.org/10.1007/978-3-658-22616-9_11

Falloon, G., Forbes, A., Stevenson, M., Bower, M., & Hatzigianni, M. (2022). STEM in the Making? Investigating STEM Learning in Junior School Makerspaces.

Research in Science Education, *52*(2), 511–537. https://doi.org/10.1007/s11165-020-09949-3

Goldin, G. A. (2002). Affect, Meta-Affect, and Mathematical Belief Structures. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A Hidden Variable in Mathematics Education?* (pp. 59–72). Springer Netherlands. https://doi.org/10.1007/0-306-47958-3_4

Greefrath, G., Hertleif, C., & Siller, H.-S. (2018). Mathematical modelling with digital tools—A quantitative study on mathematising with dynamic geometry software. *ZDM*, *50*(1), 233–244. https://doi.org/10.1007/s11858-018-0924-6

Hartmann, L.-M., & Schukajlow, S. (2021). Interest and Emotions While Solving Real-World Problems Inside and Outside the Classroom. In F. K. S. Leung, G. A. Stillman, G. Kaiser, & K. L. Wong (Eds.), *Mathematical Modelling Education in East and West* (pp. 153–163). Springer International Publishing. https://doi.org/10.1007/978-3-030-66996-6_13

Hasanah, N., Asih, T. S. N., & Kharisudin, I. (2021). Mathematical Problem Solving Skill Viewed from Epistemic Curiosity on Fostering Communities of Learners. *Unnes Journal of Mathematics Education Research*, *10*(A), 134–139.

Hendriana, H., Johanto, T., & Sumarmo, U. (2018). The role of problem-based learning to improve students' mathematical problem-solving ability and self confidence. *Journal on Mathematics Education*, *9*(2), 291–299. https://doi.org/10.22342/jme.9.2.5394.291-300

Hidayah, L., Sudarman, S. W., & Vahlia, I. (2019). Pengaruh Model Pembelajaran Reciprocal Teaching Terhadap Hasil Belajar Matematika Ditinjau dari Kemandirian Belajar Peserta didik. *AKSIOMA: Jurnal Program Studi Pendidikan Matematika*, *8*(1), Article 1. https://doi.org/10.24127/ajpm.v8i1.1925

Jablonski, S. (2024). Challenges in geometric modelling–A comparison of students' mathematization with real objects, photos, and 3D models. *Eurasia Journal of Mathematics, Science and Technology Education*, *20*(3), em2414. https://doi.org/10.29333/ejmste/14321

Kharisudin, I., & Cahyati, N. E. (2020). *Problem-solving ability using mathematical modeling strategy on model eliciting activities based on mathematics self-concept*. *1567*(3). Scopus. https://doi.org/10.1088/1742-6596/1567/3/032067

Listiawati, N., Sabon, S. S., Siswantari, Subijanto, Wibowo, S., Zulkardi, & Riyanto, B. (2023). Analysis of implementing Realistic Mathematics Education principles to enhance mathematics competence of slow learner students. *Journal on Mathematics Education*, *14*(4), Article 4. https://doi.org/10.22342/jme.v14i4.pp683-700

Magana, A. J., Arigye, J., Udosen, A., Lyon, J. A., Joshi, P., & Pienaar, E. (2024). Scaffolded team-based computational modeling and simulation projects for promoting representational competence and regulatory skills. *International Journal of STEM Education*, *11*(1), 34. https://doi.org/10.1186/s40594-024-00494-3

Maslihah, S., Waluya, S. B., Rochmad, Kartono, Karomah, N., & Iqbal, K. (2021). Increasing mathematical literacy ability and learning independence through problem-based learning model with realistic mathematic education approach. *Journal of Physics: Conference Series*, *1918*(4), 042123. https://doi.org/10.1088/1742-6596/1918/4/042123

National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics*. Reston, VA: NCTM.

National Council of Teachers of Mathematics. (2018). *Catalyzing Change in High School Mathematics: Initiating Critical Conversations*. Reston, VA: NCTM.

National Council of Teachers of Mathematics. (2020). *Standards for the Preparation of Secondary Mathematics Teachers*. Reston, VA: NCTM. https://www.nctm.org/uploadedFiles/Standards_and_Positions/NCTM%20CAEP%202020%20HS.pdf

Nurochmah, Y., & Kharisudin, I. (2023). Mathematical Modeling Problem Solving Viewed from Students' Mathematical Self-Concept on Means-Ends Analysis Based on Blended Learning. *Unnes Journal of Mathematics Education*, *12*(2), Article 2. https://doi.org/10.15294/ujme.v12i2.74003

Nurrahmawati, N., Sa'dijah, C., Sudirman, S., & Muksar, M. (2021). Assessing students' errors in mathematical translation: From symbolic to verbal and graphic representations. *International Journal of Evaluation and Research in Education (IJERE)*, *10*(1), Article 1. https://doi.org/10.11591/ijere.v10i1.20819

Rahman, A. A., & Kharisudin, I. (2019). An Analysis of Problem Solving Ability using mathematical modeling strategies in Brain-Based Learning. *Unnes Journal of Mathematics Education*, *8*(3), Article 3. https://doi.org/10.15294/ujme.v8i3.32218

Riyanto, B., Zulkardi, Z., Indra Putri, R., & Dr, D. (2019). Senior high school mathematics learning through mathematics modeling approach. *Journal on Mathematics Education*, *10*, 425–444. https://doi.org/10.22342/jme.10.3.8746.425-444

Rodriguez-Martinez, L. Y., Hernandez-Martinez, P., & Perez-Martinez, M. G. (2023). Development of a protocol to measure mathematics higher-order thinking skills in Mexican primary schools. *Journal on Mathematics Education*, *14*(4), Article 4. https://doi.org/10.22342/jme.v14i4.pp781-796

Sanjari, A., & Manouchehri, A. (2024). Interactive Learning: Unpacking the Influence of Computer Simulations on Students' Mathematical Modeling Processes. *Education Sciences*, *14*(4), Article 4. https://doi.org/10.3390/educsci14040397

Sari, N. (2024). Improving Learning Independence of Elementary Students through the Two Stay Two Stray Method. *Journal of Childhood Development*, *4*(1), 145–153. https://doi.org/10.25217/jcd.v4i1.3970

Scheiner, T., & Montes, M. A. (2024). Exploring prospective teachers' stances in making sense of students' mathematical ideas. *Journal of Mathematics Teacher Education*. https://doi.org/10.1007/s10857-024-09639-1

Serpe, A., & Frassia, M. G. (2020). *Task Mathematical Modelling Design in a Dynamic Geometry Environment: Archimedean Spiral's Algorithm* (Y. D. Sergeyev & D. E. Kvasov, Eds.; Vol. 11973, pp. 478–491). Springer International Publishing. https://doi.org/10.1007/978-3-030-39081-5_41

Su, K.-D. (2024). The Challenge and Opportunities of STEM Learning Efficacy for Living Technology Through a Transdisciplinary Problem-Based Learning Activity. *Journal of Science Education and Technology*. https://doi.org/10.1007/s10956-024-10094-z

Sutrisno, H., & Kharisudin, I. (2020). Problem solving ability with mathematical modeling strategy in term of mathematics self-efficacy on Generative Learning Model. *Unnes Journal of Mathematics Education*, *9*(1), Article 1. https://doi.org/10.15294/ujme.v9i1.35674

Villegas, J. L., Castro, E., & Gutiérrez, J. (2009). Representaciones en Resolución de Problemas: Un estudio de caso con problemas de optimización. *Electronic Journal of Research in Education Psychology*, *7*(17), Article 17. https://doi.org/10.25115/ejrep.v7i17.1342

Zulkarnaen, R. (2020). Konsepsi Siswa dalam Proses Pemodelan Matematis. *SJME (Supremum Journal of Mathematics Education)*, *4*(2), Article 2. https://doi.org/10.35706/sjme.v4i2.3638