

## **Hypothetical Learning Trajectory for Negative Integer in Differentiated Instruction: A Prospective Analysis in Didactical Design Research**

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### **Abstract**

Integers are one of the essential materials in mathematics, but provides its own difficulties for students, especially with regard to negative integers. This study aims to develop a hypothetical learning trajectory based on the results of the learning obstacle study. Specifically, the alleged learning trajectory is a conjecture on phase D students, namely Junior High School students based on the differentiation of the readiness aspects of high, medium, and low students. This study used development research with a didactical design research approach at the prospective analysis stage, namely analyzing the didactic situation before learning. The development results obtained a hypothetical learning trajectory based on the analysis of learning obstacle and literature review. Based on this, the hypothetical learning trajectory that is compiled consists of four stages starting from the concept of negative numbers, the concept of integers, counting operations on integers, and the properties of calculating operations on integers and their application. In addition to these four stages, the alleged learning trajectory also emphasizes the meaning of the minus sign as a prerequisite concept in integers. The integration of didactical situations in the hypothetical learning trajectory emphasizes the diversity of didactical situations towards students' abilities as a form of differentiated instruction, especially in differentiating content.

**Keywords:** didactical design research, hypothetical learning trajectory, negative integer, prospective analysis



## **INTRODUCTION**

Schools have the freedom and opportunity to improve learning through the Kurikulum Merdeka, a component of the Merdeka Belajar Policy. Schools must also guarantee that all students have the same rights to receive instruction following their unique needs and aptitudes through an Kurikulum Merdeka. In addition, every school is responsible for offering the resources necessary for differentiated instruction to realize each student's potential and ability fully. Differentiated instruction is a methodical approach to curriculum and instruction for students with various academic backgrounds (Tomlinson & Strickland, 2005). Students are seen as unique individuals with unique differences and as being dynamic from the perspective of differentiated instruction (Gusteti & Neviyarni, 2022). Differentiated instruction allows schools to tailor their instruction to the skills and requirements of each student (Breux & Magee, 2013; Fitra, 2022; Fox & Hoffman, 2011; Tomlinson, 2017). Differentiated instruction places a strong emphasis on three factors: student learning readiness, student interests, and student learning profiles (learning styles) (Tomlinson & Moon, 2013; Tomlinson & Strickland, 2005). The learning process is not solely centered on one learning model, approach, technique, or method, which teachers in differentiated instruction must grasp.

Aspects of students' preparation for learning are directly tied to their initial abilities, knowledge, and abilities to meet predetermined learning objectives. This demonstrates students' capacity to acquire new material, which means that an assignment based on their preparation level will push students outside of their comfort zone in a suitable and encouraging learning environment (Hockett, 2018). A teacher needs to be aware of the students' interests because they significantly impact the learning process, including student learning achievement (Purba et al., 2021). When interests are considered in the learning design, students can observe how well education aligns with their goals, identify the relationships between concepts and subjects, and use their existing understanding of abilities to pick up new ones (Tomlinson, 2001). The technique or approach that students prefer to grasp learning is called the student learning profile (learning style). The teacher will carry out three crucial diversification-related tasks in a classroom where students are receiving differentiated instruction: differentiation in terms of content, process, and product (Tomlinson & Moon, 2013).

Content elements refer to the material taught by the teacher or studied by students in the classroom. Differentiated instruction requires teachers to produce lesson materials suited to students' levels of readiness and interest, depending on each student's profile or learning style (Purba et al., 2021). Process elements are activities or tasks students perform during the learning process in class. The learning activity in issue is an activity or activity that is relevant and associated with the content being studied and where a qualitative assessment is offered to students in the form of feedback notes (Purba et al., 2021). Process components, like content elements, should be based on student preparation, interests, and learning profiles. Moreover, the product aspect is the results of learning that can represent students' abilities, knowledge, and skills after completing a task or following a lesson. In this sense, the product must be differentiated based on the student's readiness, interests, and learning profile.

Integers is a material in mathematics that junior high school students must learn. For junior high school students (Phase D) in the Kurikulum Merdeka, one of the Mathematics Learning Outcomes (MLO) is in the number domain, particularly in integers, meaning that students can read, write, and compare integers, including algebraic operations on integers. Due to its connections to other mathematical ideas, particularly those connected to arithmetic, integer material is among the most crucial resources in learning mathematics (Akyüz et al., 2012; Peled et al., 1989). One of the realms covered in all other mathematical notions is the concept of numbers, especially integers, and operations on numbers. Mathematics problem-solving is based on the idea of integers and the algebraic operations that apply to them. Therefore, students can necessarily comprehend integers and related operations.

Students who use reasoning to represent a number less than zero without concrete modeling begin their study of numbers with integers (Stephan & Akyuz, 2012). The set of integers comprises the zero, the set of negative integers, and the set of positive integers. One of the subjects that students frequently need help to acquire is integers and operations on integers (Kilhamn, 2011; Vlassis, 2008). In addition, mastering negative numbers in particular poses unique challenges for students (Bishop et al., 2014; Blair et al., 2012; Larsen, 2012). Similar issues are encountered by students in Indonesia as well, where they continue to run into difficulties when learning the content on integers, which negatively affects their learning achievement in this subject (Fuadiah et al., 2017b, 2019a; Subanji, 2015). One of the presentations of the concept of integers in seventh grade VII mathematics textbooks in the Merdeka Curriculum begins with the definition of integers, then declaratively presents the set of parts of integers, namely positive, negative, and zero integers. However, specifically, in negative integers, the concept of the minus sign is not made part of the concepts taught to students. This will potentially create obstacles, especially when students learn arithmetic operations on integers, especially the subtraction operation of negative integers. Thus, the calculation operations of negative integers are only presented in a declarative manner.

It is crucial to anticipate the difficulties that students will encounter when studying the concept of integers to improve their knowledge in the future. The learning process is one of the crucial factors that must be considered so that student learning becomes essential in learning activities (Schmidt, 1997). This shows that the teacher's lesson design needs to be truly flexible to the demands or circumstances of each student, including their unique (differentiated) abilities or needs. One way that can be done is to set up a learning trajectory or flow suitable for the students' level of thinking. Teachers should pay close attention to their students' learning trajectories when creating learning plans (Daro et al., 2011). This will enable the learning plans to be developed around the expectation of all potential student learning developments. Therefore, teachers must pay attention to students' thinking levels and the learning process, also known as the Learning Trajectory (LT) (Clements & Sarama, 2004).

Clements and Sarama (2004) define learning trajectory as a description of students' thinking processes in the learning process, in the form of conjectures or hypotheses from a series of learning designs to develop students' abilities in order to achieve learning goals.



Simon (1995) stated the phrase "*learning trajectory*" to describe the learning process. The main concern should be the teaching methods used by teachers and the information that students can obtain about how to think about mathematical concepts, including how to create a learning experience that can help students understand mathematics through the learning plans created (Simon, 1995). The teacher will actively participate in the interaction process with students to directly monitor the students' learning process, such as what students grasp and how students apply ideas to solve issues. Teachers must didactical situation in which students' mathematical concepts can be stimulated through inquisitive activities (Brousseau, 2002). This shows that learning designs are growing more sophisticated because they will consider the diversity of student responses so that student responses can impact the following series of didactic situations. Thus, the teacher's learning trajectory must accommodate various students' needs or abilities. As a result, a differentiated instruction approach serves as a bridge in addressing this variability by aligning with estimating the responses that arise.

In its development, the continuity of students' learning must consider the consistency of their thought processes (Clements & Sarama, 2011). According to Simon, the learning trajectory consists of three components: learning objectives, actual learning activities, and hypotheses answers that will be made about thinking and learning involving students (Clements & Sarama, 2004). Following learning experiences and the range of mental objects generated, Suryadi (2019) claims that each learning trajectory is a distinct instrument for achieving learning goals. It was further explained that in order to facilitate this diversity, teachers needed to be able to anticipate different possible student reactions as a result of the didactic situations that were set up so that the purported learning path (Hypothetical Learning Trajectory/HLT) could serve as a guide for the facilitation and intervention processes that would be carried out. HLT describes the learning process that includes learning objectives, tasks that students complete, and estimations of their thinking and learning (Simon, 1995). Therefore, HLT gives teachers a summary of a learning plan based on students' anticipated learning to accomplish learning objectives.

## **METHODS**

This research was developmental research that used the didactical design research paradigm at the prospective analysis stage which aims to develop a hypothetical learning trajectory for negative integer based on student readiness in differentiated learning. The prospective analysis stage aims to analyze the didactical situation before learning in the form of hypothetical didactical design, including hypothetical learning trajectory (Suryadi, 2019). The results of a literature review analysis, which included a study of learning obstacle, a curriculum study based on integer content for junior high schools, and students' thinking phases, were used to create the hypothetical learning trajectory. The preparation of the hypothetical learning trajectory is based on the learning objectives for mathematics, the learning activities that support the learning objectives, and the anticipated thinking of the students following their abilities (Simon, 1995). The steps in

this research are an adaptation of Fuadiah's stages of HLT preparation (Fuadiah, 2017) as follows in Figure 1.

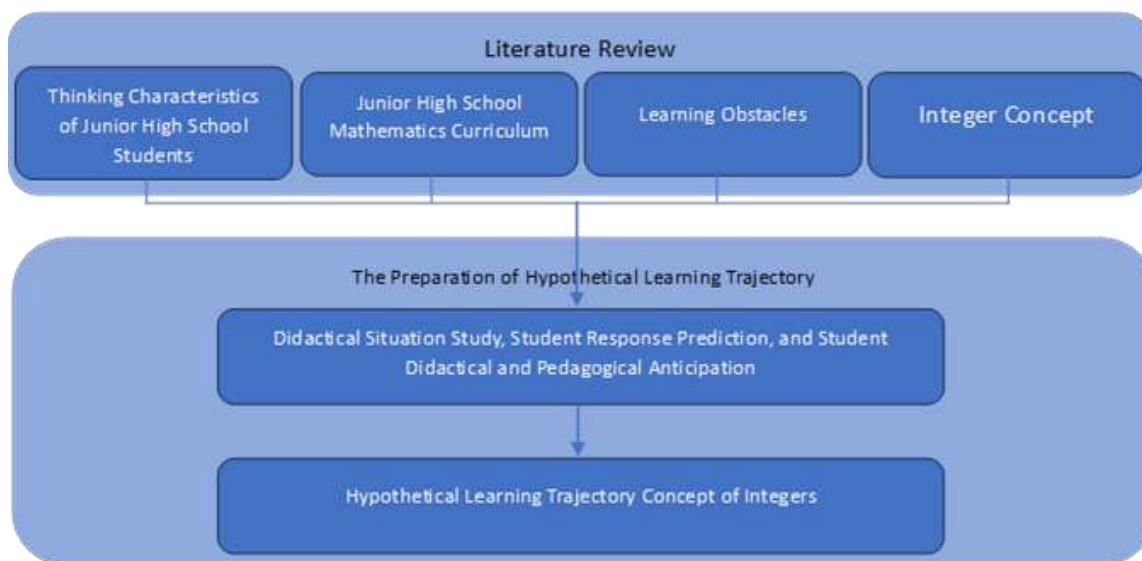


Figure 1. The Preparatory Stages of the Hypothetical Learning Trajectory

In the first stage of Figure 1, a literature review was conducted in the form of student characteristics, curriculum, learning obstacles, and the concept of Integers to obtain various information or data needed as the basis for the preparation of the hypothetical learning trajectory. The first literature review conducted was related to the thinking characteristics of junior high school students in general, including learning styles. Furthermore, a literature review was conducted related to the Junior High School Mathematics Curriculum in the Kurikulum Merdeka, especially for Integers. In this case, a study was carried out regarding learning outcomes through curriculum documents, then elaborated in detail into a flow of learning objectives as a basis for preparing didactical situations. Another literature review is related to learning obstacles which aims to find out what obstacles or mistakes students experience in learning or solving Integer problems. This study needs to be done in order to avoid the potential repetition of the same obstacles or errors. Thus, the resulting didactical situation is expected to reduce learning obstacles and not be imitative for the emergence of similar potential learning obstacles. The last literature review is related to the concept of Integers including the researches that have been done related to Integers which aims to obtain the concept used as a reference in developing the didactical situation on the hypothetical learning trajectory.

In the second stage, with reference to the results of the study in the first stage, the preparation of the hypothetical learning trajectory was carried out including reviewing and developing didactical situations to be built. In preparing the didactical situations integrated in the hypothetical learning trajectory, the students' thinking stages refer to Gravemeijer (1998) statement that there are four stages of students' thinking called conceptual learning trajectory levels, namely: (1) situational level, where students are in a specific context; (2) referential level, which is a strategy or model that fits the situation or problem; (3) general level, which focuses on mastering strategies that refer to the

context; and (4) formal level, which is the stage of working on conventional procedures and notations. The results of the development of the hypothetical learning trajectory including the didactical situations developed were then discussed with colleagues as a form of data credibility test in the data validity test.

## RESULTS AND DISCUSSION

In the Kurikulum Merdeka, in Phase D of the numbers element, the material pertaining to integers is offered to junior high school students in class VII. Following the Decree of the Head of the Educational Standards, Curriculum, and Assessment Agency of the Ministry of Education and Culture Number 008/H/KR/2022, the Study of Numbers as Number Symbols, Number Concept, Number Counting Operations, and the Relationship between Different Number Counting Operations in the Sub-Elements of Visual Representation, Properties Sequence, and Operations are included in the Study of Numbers element. Phase D presents integers, and at its conclusion, it lists the following as the learning accomplishments for the number element.

Table 1. Learning Achievements of Phase D Number Elements

Element	Learning Achievements
Number	<ol style="list-style-type: none"><li>1. Students can read, write, compare, and use exponents, roots, rational and irrational numbers, decimal numbers, rational and irrational numbers, and numbers in scientific notation</li><li>2. Students can solve problems (mainly financial literacy) by applying arithmetic operations to actual quantities and giving estimates or approximations</li><li>3. Students can utilize prime factorization and knowledge of ratios (scale, proportion, and rate of change) to resolve problems</li></ol>

Furthermore, by concentrating solely on integers, phase D's targeted learning outcomes include enabling students to read, write, and compare integers and perform arithmetic operations (calculation operations). It is also explained in the Learning Flow and Objectives (*Alur Tujuan Pembelajaran/ATP*) for Whole Numbers for Class VII Junior High School Students by referring to the defined learning goals. The ATP in question often covers integers concepts, including positive, negative, and zero, as well as arithmetic operations on integers and how to use integers to solve problems in context.

### Hypothetical Learning Trajectory for Integers

Before developing the hypothetical learning trajectory, a literature review was conducted related to learning obstacles through several research results. The findings of studies on integers, particularly in Indonesia, demonstrate that students often struggle to comprehend the material, which lowers their understanding of the subject (Fuadiah et al., 2017b, 2019a, 2019b; Subanji, 2015). In this study, it was discovered that students had learning obstacle on integers. According to Fuadiah et al. (2017a), students face several learning obstacle when learning negative integer-related ideas, procedures, principles, and problem-solving techniques. Meanwhile, Fuadiah et al. (2017a) discovered that

students encountered learning challenges related to necessary information. In a number of the texts that students use to study integers, there is a leap in comprehension whereby students become familiar with the idea of numbers right away. This could present problems for them.

As Fuadiah et al. (2017a) point out, when it comes to conceptual learning obstacle, students' knowledge of the meaning of integers, the order in which they appear, and how they are represented are the critical areas of concern. Regarding the idea of integers, this is a fundamental notion that students should be able to grasp. They should know what an integer is, how to arrange them, and how to represent them in real-world situations. Solving problems with integers becomes increasingly challenging due to the learning obstacle in these fundamental ideas (Schmidt, 1997). The results of the study showed the existence of learning obstacles in students so that it has an impact on the low ability of students' understanding of the concept of integers. Researchers put special emphasis on the concept of negative numbers and found a series of learning obstacles experienced by students in the aspects of concepts, procedures, principles, and applications in solving problems related to negative numbers. With reference to this, a draft hypothetical learning trajectory was prepared by referring to the content of the integer material in accordance with the Kurikulum Merdeka with an emphasis on the material according to the identification of learning obstacles.

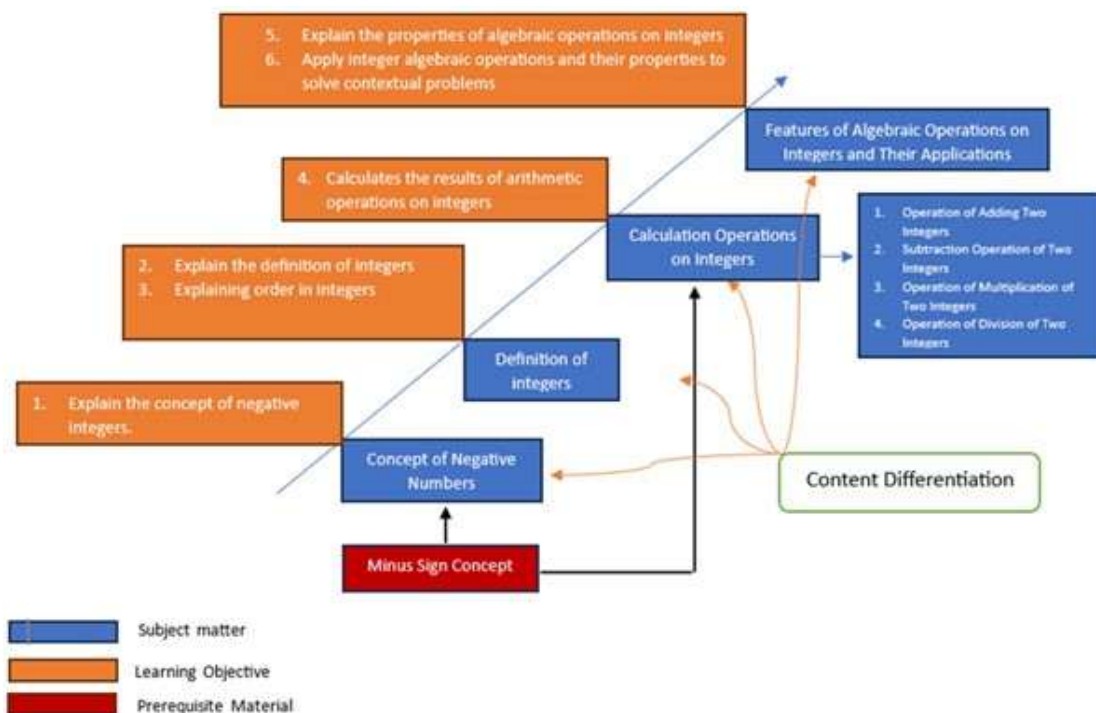


Figure 2. Hypothetical Learning Trajectory of Integers

Figure 2 above shows that the conjecture of students' thinking to learn the concept of whole numbers is broadly divided into four main parts, namely the concept of negative numbers, the definition of whole numbers, whole number operations, and the properties of whole number operations and their applications. In addition, Figure 2 also presents one

prerequisite material, namely the minus sign. Furthermore, by paying special attention to the learning obstacle study and the study of the concept of Integers, it shows that one of the crucial things that need to be mastered by students as well as a frequent Learning Obstacle is related to the concept of negative numbers. Therefore, in this study, special emphasis is given to negative numbers as a concept that will be further developed into a didactical situation as an integration of the hypothetical learning trajectory in Figure 3.

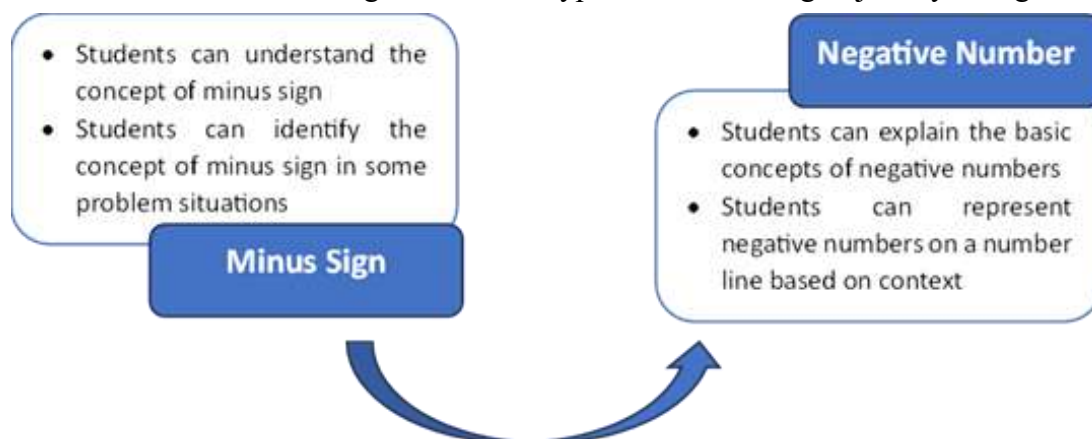


Figure 3. Hypothetical Learning Trajectory of Negative Numbers

The introduction of the idea of negative numbers is where the purported learning flow for the class starts. Students were already familiar with integers and natural numbers from the previous session. With the help of these two ideas, students will be encouraged to comprehend the idea of negative numbers by linking them to natural numbers. Eventually, they will be able to identify negative integers. This is crucial because, according to several research findings (Fuadiah et al., 2017b, 2019b, 2019a; Subanji, 2015), negative number, especially negative integers, are more problematic for pupils than positive ones. To do this, the purported learning flow will initially concentrate on negative numbers by teaching students about the minus sign.

It is critical to accurately write mathematical symbols in mathematics to comprehend the intended meaning. According to Bofferding (2014), Lamb et al. (2012), and Vlassis (2008), there are three possible interpretations for the "-" symbol or minus sign in Table 2.

Table 2. Meaning of Minus Sign

Meaning of Minus Sign	Problem Example
Subtraction – Binary Function	$2 - 3 = \dots$
The Symbol for Negative Numbers – Unary Function	$-2$
Opposite of – Symmetric Function	Which one is greater, $- -4$ or $-4$

Table 2 shows that the minus sign has three possible meanings, namely as a binary function of subtraction, indicating a negative number, and the opposite of a number (opposite). By referring to Table 2, the design of the didactical situation as presented in Table 3 at the situational level presented refers to the three meanings or meanings of the minus sign, either functioning as unary, binary, or symmetry. Thus, students' understanding of the meaning of the minus sign is not only as a subtraction operation (binary function),

but can also be as an uner or symmetrical function. The difference in meaning provides flexibility in use according to the context at hand. The three minus signs need to be well understood by students, so that when learning integer concepts, such as order and counting operations, they will not be mistaken. Thus, when students are faced with the minus sign, they can correctly understand the meaning of the sign according to the context of the problem at hand, especially in relation to negative numbers.

In the concept of integers, negative numbers in integers are a significant concern that students must understand (Fuadiah et al., 2017b, 2019a, 2019b; Subanji, 2015). However, they struggle to master negative numbers in mathematics education (Bellamy in Fuadiah et al., 2017a; Vlassis, 2008). Despite this, other papers, like those by Hefendehl-Hebeker in 1991 and Gallardo in 2002, demonstrate that mathematicians must work hard to develop the concept of integers, particularly negative integers (Fuadiah et al., 2017a). Negative numbers have a higher error rate among pupils since they are tied to the concept of the minus sign. Thus, this study's primary focus is establishing didactical situations with negative numbers.

### **Integration of Didactical Situations in Hypothetical Learning Trajectory**

The results of the learning obstacle study show that there are still student obstacles when faced with whole number operations involving the minus sign. The concept of minus sign is a crucial thing that needs to be mastered by students in learning negative numbers. The minus sign denoted by "-" can have three functions, namely uner, binary, and symmetric functions. Therefore, in anticipating the potential for learning obstacles to occur again, especially in integer operations involving the minus sign, students must first master the concept of the minus sign and be able to identify the minus sign in various problem contexts, namely whether it functions as uner, binary, or symmetrical.

Some didactical situation can be created while teaching students about the negative signs as in Table 3 below.

Table 3. The Design of Minus Sign Didactical Situation

<b>Level</b>	<b>Design of Didactical Situations</b>
Situational	The opposite, binary, and unary operations of the negative sign and other contextual issues are given. For example: <ol style="list-style-type: none"><li>1. Budi has eight candies to share with a friend. How many candies does Budi have if his friend receives five?</li><li>2. Which value between <math>-1</math> and <math>-(-1)</math> is smaller?</li><li>3. In the number line game, a student advances five steps backward. How to write five steps backward?</li></ol>
Referential	Develop problem-solving strategies or a series of questions/statements related to the given problem
General	Identify the use of the minus sign
Formal	Apply the minus sign concept to mathematical problems



The preparation of the didactic situation design refers to each student's diverse abilities and situations directly tied to the student's life or are close to the student's environment in Surakarta city. Thus, a specific idea's didactic situation comprises several situations based on the students' abilities as in Table 4 below.

Table 4. The Design of Didactical Situations Concept of Negative Numbers

Level	Design of Didactical Situations
Situational	<p>Various contextual problems related to introducing negative numbers are presented according to students' abilities.</p> <p><b>Alternative Didactical Situations 1-High Mathematical Ability</b></p> <p>The first contextual problem concerns placing things on the river's surface. In this context, problems are designed by incorporating river information that students already know, such as the Bengawan Solo River in Surakarta, which is one of the longest rivers on the island of Java, stretching 600 kilometers from the Sewu mountain to the Java Sea north of Surabaya with a depth ranging from 0.3 to 8 meters. The flow of the Bengawan Solo River is frequently exploited to meet daily necessities like irrigation; hence, dams have been built in numerous locations, including the Colo Dam. The water surface height is one of the data that can be gathered from the dam using the available measuring tools.</p> <ol style="list-style-type: none"> <li>1. If an officer tosses an object into a river, object A is 5 meters below the water's surface, and object B is 2 meters above the water's surface, sketch the problem!</li> <li>2. Draw a diagram of the problem if two things float beneath the sea's surface: object C is 2 meters deep, and object D is 7 meters deep. Which object is the farthest from the water's surface?</li> <li>3. Using the two pictures from questions 1 and 2, present the sequence of object placements on the number line if the sea surface displays the number 0!</li> </ol> <p><b>Alternative Didactical Situations 2-Medium Mathematical Ability</b></p> <p>On November 14, 2015, the Solo city government announced plans to build a two-story parking structure beneath West City Square on the Solopos Soloraya online news page. It was also stated that the parking garage had a basement system.</p> <ol style="list-style-type: none"> <li>1. If two visitors, A and B, enter the parking building, and the parking officer advises A to park on the floor under West City Square and B on the floor under West City Square, depict the problem below in sketch form!</li> <li>2. If three more parking lots are added to the parking lot above, and a visitor, C, is directed to park on the second floor of the lot, illustrate the problem in sketch form!</li> <li>3. Considering the two illustrations from problems 1 and 2, represent the order of the three visitors' parking spaces on a number line if the position of the West City Square shows 0 on the number line!</li> </ol> <p><b>Alternative Didactical Situations 3-Low Mathematical Ability</b></p> <p>Problem situations involving the number line can be presented directly (this can be done in class utilizing the classroom floor marked with positive, zero, and negative numbers).</p> <div style="text-align: center;"> </div> <p>The problem can be resolved by placing students in positions on a number line. For example, a student has five units to the right of 0 and another five units to the left of 0.</p>
Referential	Develop problem-solving strategies or a series of questions/statements related to the given problem
General	Identify the use of the negative numbers
Formal	Apply the negative concept to mathematical problems

Forming didactical situations relating to negative numbers in Table 4 is a critical topic or issue in studying integers. The didactic situation consists of two parts: (1) determining the position of objects/objects corresponding to the presented conditions; and (2) comparing the positions of objects/objects on a number line. The first, second, and third alternative situations assist students with high, medium, and low student skills. These conditions allow for content differentiation through learning situations or activity differentiation (Purba et al., 2021). The learning setting requires students to explain a problem by placing an object on a number line, notably positive and negative numbers. Suryadi (2019) claims that, in this case, a critical question must be asked of students for the didactic situation to be meaningful.

In Table 4, students' knowledge begins in the prepared didactic environment by offering a contextual problem circumstance. Choosing the appropriate context for the material content is critical for increasing student understanding (Fuadiah et al., 2017b). The first alternative situation introduces context about an object's position in and above the water. The water surface is a boundary-indicating condition. Furthermore, the second alternative circumstance is constructed similarly to the first, except that the context is tied to structures below and above the ground surface. Because they feature problem settings that do not directly use a number line, these two alternative situations correspond to content differentiation directed at students with high or medium skills. The issue asks students to draw a number line showing the positions of positive and negative numbers. According to Fuadiah (2017), this represents a number line concerning negative, zero, and positive values. The number line is a model that assists students in simplifying numerical arithmetic processes so that they can answer issues (Altiparmak & Ozdogan, 2010). As a result, it is hoped that students will grasp the concept of negative numbers and order in negative and positive numbers. On the other hand, the role of the teacher in learning is also essential because the teacher not only supervises learning activities but also interacts with students, including asking critical questions, as Suryadi (2019) mentioned.

The steps at the referential level are critical in didactic situations because students are urged to abstract contextual problems so that they can solve problems using techniques or models suitable to the context of the problem. The expected illustration for pupils with high and medium ability is a number line with positive, zero, and negative values. The object to the left of zero is the inverse of the thing to the right of zero. A positive number appears when the object advances to the right and vice versa. However, teachers must keep in mind that these conditions allow for inquiry, which can affect errors in grasping integers (Fuadiah et al., 2019b).

The notion of integers can be built by integrating knowledge of negative, zero, and positive integers. Negative numbers, including integers, are essential to examine in learning integers. It will be tied to arithmetic operations on integers involving negative numbers. Arithmetic operations involving negative numbers, such as addition, subtraction, multiplication, and division, require greater attention.

Integer operations begin with addition and subtraction. According to Fuadiah et al. (2017a), learning integer operations is divided into two stages, the first of which consists



of solving simple integer-related issues and the second of which consists of calculating integer operations. According to this viewpoint, didactic scenarios are created by providing simple issue situations such as simple addition or subtraction of whole numbers. Besides, a problem might be constructed using a number line in comprehending the concept of arithmetic operations on integers. Beswick adds that arithmetic operations on integers, including addition and subtraction, can be represented on a number line (Beswick, 2011). When adding and subtracting two positive integers, the didactic situation can be set up so that the representation on the number line can be expressed as a shift of an object, specifically when addition shows a shift to the right from zero and subtraction shows a shift in the opposite direction, allowing the operation to be understood with the words "continued with." Therefore, the problem  $a + b$  or  $a - b$  shows the overall position of the object from zero (Figure 4 and Figure 5). Students can understand one of the integer operations  $a - b = a + (-b)$  with this representation. The setting that can be offered is a student ascending or descending the stairs at home.

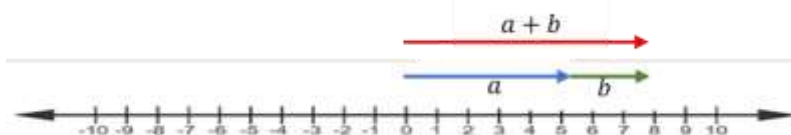


Figure 4. Representation of the Adding Operation of Two Positive Integers

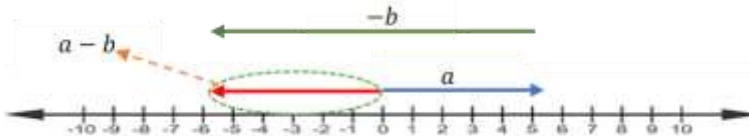


Figure 5. Representation of the Subtraction Operation of Two Positive Integers

Subtraction of two negative numbers is the major problem with arithmetic operations on primary relevant integers. Considering the claims made by Bofferding (2014), Lamb et al. (2012), and Vlassis (2008) outline that the opposite of the second negative integer being subtracted can be deduced from the subtraction process using the minus sign. Thus, the applicable property is  $a - (-b) = a + b$ , where the sign minus can be interpreted as the opposite of the number  $-b$ , namely  $b$ . The representation on the number line can be stated as follows in Figure 6.

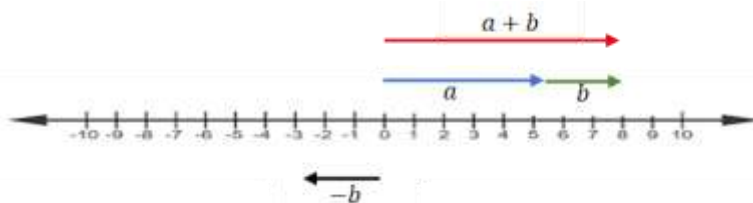


Figure 6. Representation of Negative Integer Subtraction Operations

Different contexts corresponding to students' abilities can be created by organizing didactical situations in differentiated instruction. However, the fundamental idea behind these situations is the concept of integers; thus, with the variety of contexts, learning is meaningful for all levels of student ability.

## CONCLUSION

Integers are a fundamental concept that is essential in learning mathematics. Students' understanding of the concept of whole numbers is still unsatisfactory, where students still experience difficulties when dealing with negative whole numbers. In addition, there is diversity in terms of students' abilities. Hypothetical learning trajectory was developed based on learning obstacles including concept gaps in the references used in learning and literature review related to the concept of integers. The hypothetical learning trajectory consists of learning stages related to the concepts of negative numbers, whole numbers, counting operations, and their applications. In addition, this hypothetical learning trajectory also emphasizes the meaning of the minus sign as a prerequisite concept. The integration of didactical situations in the learning trajectory is presented in a variety of content so that it is expected to facilitate the diversity of student abilities in differentiated learning.

The research conducted is limited to developing a hypothetical learning trajectory. It would be very interesting if in future research the implementation of the hypothetical learning trajectory is carried out so that it can be seen how the hypothetical learning trajectory plays a role in minimizing learning obstacles that could potentially occur to students, including how effective it is for the diversity of student abilities.

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