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Newman Error Analysis (NEA): Identifying Students' Functional Thinking Errors Based on Self-Efficacy

Suci Yuniati [1]*, Rizky Aulia Nisa[2], Annisah Kurniati[3], Depriwana Rahmi[4], Evrim Erbilgin [5]

[1,2,3,4] Program Studi Pendidikan Matematika Fakultas Tarbiyah dan Keguruan, Universitas Islam Negeri Sultan Syarif Kasim Riau, Indonesia.

[5] The Pedagogy Technology and Teacher Education Division, Emirates College for Advanced Education, Abu Dhabi, United Arab Emirates.

*Corresponding Author. E-mail: suci.yuniati@uin-suska.ac.id [1]

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Abstract

This study aims to analyze the relationship between self-efficacy and high school students' errors in functional thinking using Newman Error Analysis (NEA). This study uses a qualitative approach with a descriptive method to analyze the pattern of students' functional thinking errors in solving mathematical problems. The subjects of the study were selected by purposive sampling based on variations in students' self-efficacy levels (high, medium, and low) to obtain a more comprehensive representation in the analysis of functional thinking errors. Students' self-efficacy was measured using a Likert scale-based questionnaire, while essay-based tests were used to identify their mathematical functional thinking abilities. Additional data were obtained through interviews based on interview guidelines to understand students' functional thinking processes in solving problems. The validity of the research data was achieved using the triangulation method by comparing test result data, questionnaires and interview guidelines. The results showed that students with high and medium self-efficacy tended to make fewer functional thinking errors than students with low self-efficacy. Students with low self-efficacy made more errors in understanding problems (comprehension errors), transforming information (transformation errors), applying procedures (process skill errors), and writing final answers (encoding errors). These findings can be a basis for teachers in designing more effective learning strategies to improve understanding of the concept of function.

Keywords: NEA, functional thinking, self-efficacy, problem solving

INTRODUCTION

Mathematics is a science that relies on the ability to solve problems related to coherent thinking. As the result, it develops students' thinking abilities. In solving math problems, there are some general steps to take, namely, (1) understanding the content of the math problem by providing mathematical symbols, (2) solving it using mathematical language, and (3) arriving at the completion stage (Prayitno, et al., 2022; Al Farra, et al., 2022). However, in solving math problems, many students find themselves make errors due to their failures in the the first stage (not understanding the math concept). Thus, in identifying student errors, math teachers are suggested that they should have not seen from the final answer alone, but also examined the underlying process of reaching the answers. When the math teachers know the sources of the students' errors, they can show them where their mistakes are and the students can correct themselves independently. In the future, hopefully they do not make the same errors again.

Errors can be used as the stepping stones to help students to study better, but when they are not well clarified to the students, on the other hand, they can be the stumbling blocks that break enthusiasm to move forward and prevent knowledge from developing well. Indeed, encountering errors means a sign to run an evaluation (Paladang et al., 2018). Evaluation is carried out by looking more deeply at the errors made by the students and investigating the factors that cause these errors. In other words, an in-depth analysis for each identified error is needed.

One of the approaches to analyze mathematic functional thinking errors is Newman Error Analysis (NEA). Up to now, NEA is a widely used method by the researchers. This method was first introduced by M. Anne Newman in 1977. Newman specifically defined five skills in solving mathematics, namely: reading, understanding, transformation, processing, and encoding (Tekaeni et al., 2020) and Chiphambo & Mtsi, 2021). When the students fail to attend to one of them, they will likely face some math problems. For example, when the students cannot correctly read the problem, then reading errors occur thus the students cannot correctly point the mathematical symbols or notation. It happens many times that the students cannot state what is known and what is asked from a question. In another time, the errors take place during transformation stage that is when the students cannot mention the formula or calculation that is required by the question. For instance, an error of transformation stage is when the students cannot recall the arithmetic operations or calculation steps to solve the math problem. Meanwhile, another type of error also happens when the students cannot write the conclusion as the final answer to the question.

Errors that occur due to the students' weak learning processes. For instance, during the learning process in class, the teachers do not provide enough stimulations to the students to develop their thinking skills. On the other hand, the learning process is more directed at developing students' ability to memorize information. In reality, many mathematics learnings in schools solely focus on results of thinking and less to the thinking process itself. Thinking is a human personal activity that leads in discoveries directed towards particular goals, among them is finding the desired understanding (Paladang et al., 2018). As reviewed by Paladang et al., (2018), thinking allows a process that produce mental representations via simulation of information transformation that interact with complex mental attributes, such as judgment, abstraction, reasoning, imagination, and problem solving. Hence, developing students' thinking processes in learning mathematics is paramount, so that they can use their thinking skills in understanding mathematics material as expected and gradually minimize the difficulties they face (Kamarulzaman et al., 2022).

Students' thinking in general are divided into two types, predicate and functional thinking. Functional thinking is very necessary in learning mathematics because it is one of prerequisite skills to algebraic thinking, that includes generalization functions. Experts define functional thinking as a type of thinking that examine the relationship between two or more variants producing a function (Allday, 2018; Blanton et al., 2015; Stephens et al., 2017; Warren et al., 2013; Wilkie & Clarke, 2015; Yuniati, 2022). One way that can be used to exercise functional thinking skills is through frequently solving mathematical problems. When the students are accustomed with the process of problem solving when working on mathematic problems, it is hoped that the flow of thinking can be seen. The flow of students' functional thinking processes can be measured by referring to the indicators used by the experts, namely determining recursive patterns, determining covariational relationships and determining correspondence (Blanton et al., 2016; Tanişli, 2011; Warren & Cooper, 2005; Wilkie, 2015). When students think functionally, there are many aspects involved, one of them is the affective aspect. The affective aspect of learning mathematics includes interest, attitudes, appreciation, abilities and creativities that are shown during the learning process. Another aspect of the affectives is self-efficacy. Self-efficacy is an attitude that ones hold toward themselves upon assessing or considering their own abilities in completing specific tasks (Hendriana, H., et al., 2021) and (Lestari, et al, 2018). The indicators of self-efficacy are having confidence in: 1) one's own abilities, 2) adapting and facing difficult tasks. 3) encountering challenges. 4) completing specific tasks. 5) completing different tasks (Paladang et al., 2018).

However, many studies have been conducted related to functional thinking, including students' functional thinking processes in solving mathematical problems based on APOS theory, that concludes students' partial functional thinking consists of several stages: 1) problem identification, 2) organizing data, 3) determining recursive pattern, 4) determining the covariance relationship, 5) generalizing the relationship between variations in quantity (correspondence), and 6) re-checking the results of the generalizations of the relationship between variations in function forms carried out partially using arithmetic formulas (Yuniati et al., 2020). Another research also reports that the functional thinking process demonstrated by Junior High School students in solving mathematics problems shows that the first subject's thinking process is partial, whereas the regularity of patterns is identified in each part, while the second subject's thinking process is inductive, whereas the the thinking process is expanded according to what is asked in the question, namely by applying the method that has been obtained for t table (Suryowati, 2021). Some research findings showed that the students' functional thinking in the completion process used multiple representations and the students were able to generalize using arithmetic sequence formulas (Yuniati, et al,. 2020; Yuniati et al., 2019). Specifically, three levels of functional thinking were discussed: 1) maintenance of function, 2) deficit determination, and 3) intervention selection (Allday, 2018). Previously, research on students' functional thinking that used three variables and focused the impacts on completion of the composition function was carried out in Indonesia (Yuniati, 2022). While studies have examined functional thinking in various contexts (Yuniati, et al.,2022; Suryowati, 2021), few have explored its relationship with self-efficacy. Moreover, the application of NEA in analyzing functional thinking errors remains underexplored. This study aims to fill this gap by examining how students' self-efficacy levels influence their functional thinking errors through the NEA framework.

Aim of the Study

Many previous studies only focus on the students' mathematical abilities without detecting the causes of the students' errors. Meanwhile, in the field there are more students who have wrong answer sheets than the correct ones. Thus, there is a need to conduct research that reveals students' errors in working on mathematical problems, especially students' functional thinking and self-efficacy. Therefore, the aims of this research are, analyze the relationship between self-efficacy and high school students' errors in functional thinking using Newman Error Analysis (NEA).

METHODS

Research Design

This study employs a qualitative research approach with a descriptive case study design. Qualitative research, as defined by Creswell (2012), possesses several characteristics: 1) examines natural conditions where the researcher serves as the key instrument for data collection, 2) Data are collected in the form of words, images, or observations rather than numerical data, 3) Emphasis is placed on processes rather than merely on outcomes, 4) Data analysis follows an inductive approach, and 5) Meaning is prioritized in data interpretation. Considering these characteristics, a case study design was chosen as it allows an in-depth investigation of students' functional thinking errors in mathematical problem-solving. (Creswell, 2012) describes case study research as a method of collecting and analyzing data related to real-life cases that involve challenges, difficulties, or obstacles encountered by individuals or groups. In this study, the case study approach facilitates a detailed examination of how self-efficacy influences students' functional thinking and mathematical errors based on NEA.

Setting and Participants

This research was carried out at SMA PGRI Pekanbaru, totaling 36 students. In taking samples, researchers used purposive sampling. Purposive sampling is a technique for collecting data sources by choosing those who have the data that the research aims (Creswell, 2012). The research subjects were 3 students, namely one student with the high self-efficacy category, one student with the medium self-efficacy category, and one student with the low self-efficacy category. Research subjects were selected based on the results of the self-efficacy questionnaire and NEA.

Instruments

Data collection instruments include a questionnaire, test and interview. The self-efficacy questionnaire was designed based on the mathematical self-efficacy indicators. Meanwhile, the test was designed in the form of descriptive questions based on functional thinking indicators. The interview was prepared based on functional thinking indicators and NEA. The research instruments were validated by a panel of three experts: One mathematics education expert, One mathematics expert, and One mathematics teacher. The instruments were deemed valid when all experts confirmed their appropriateness and provided their approval for use. The functional thinking test questions used in this research are presented in Figure 1.

Look at the image below!

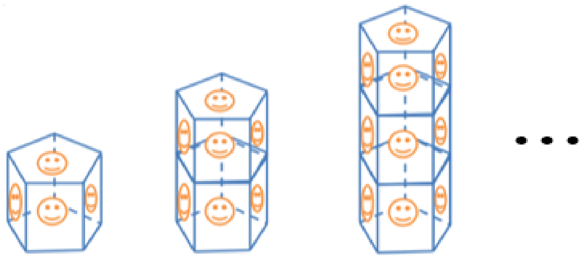


Figure 1 Figure 2 Figure 3

Instruction: for each side of the pentagon prism has a sticker
Determine:
a. Number pattern formed from the number of stickers on each side?
b. The relationship between the number of pentagonal prisms and the number of stickers?
c. General formula for drawing if there are n pentagonal prisms that will be given stickers?

Figure 1. Functional Thinking Test Questions

Data Analysis

In qualitative research, data are obtained from multiple sources using various collection techniques and analyzed continuously until data saturation is reached. This study followed three stages of qualitative data analysis, adapted from (Creswell, 2012):

Data Reduction

The researchers collected, summarized, and selected relevant data to answer the research questions. The steps in this phase included: 1) Classifying students into three self-efficacy categories based on questionnaire results, 2) Identifying functional thinking errors based on NEA, 3) Using questionnaire and test results to formulate interview questions aimed at understanding students' cognitive processes.

Data Organization & Categorization

The data were organized into three main categories: 1) Self-efficacy questionnaire results, 2) Functional thinking test results, and 3) Interview transcripts. This categorization facilitated an in-depth thematic analysis of students' mathematical errors.

Conclusion Drawing & Verification

Researchers synthesized findings from multiple data sources to answer the research questions. The final analysis provided insights into how students' self-efficacy levels influenced their functional thinking errors.

Triangulation for Data Validity

To ensure the credibility and reliability of the findings, this study employed triangulation. According to (Creswell, 2012) triangulation is a method of cross-verifying data from multiple sources to strengthen research validity. In this study, triangulation was achieved by: 1) Comparing students’ responses in the questionnaires, functional thinking tests, and interviews. 2) Identifying consistencies and discrepancies between different data sources. 3) Validating interpretations by cross-referencing responses from different participants. This process ensured that the data were accurate, reliable, and reflective of students’ actual problem-solving processes.

RESULTS AND DISCUSSION

The researchers carryied out the research by obtaining permission from the school authorities. After obtaining permission, the researchers then coordinated with the class XI mathematics teacher to determine further research schedule. Next, in the appointed time, the researchers administered the functional thinking test questions and self-efficacy questionnaires to 36 class XI Science students with a time duration of 2×45 minutes. Then the researcher collected the questionnaire and examined the results. The results of self-efficacy questionnaire is presented in Table 1.

Table 1. Self-Efficacy Questionnaire Score Interval

Score	Frequency	Category
30-70	2	Low
71-110	29	Medium
111-150	5	High

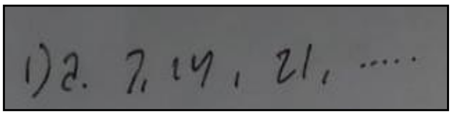
Based on data in Table 1, the researchers then took 3 students who would represent different self-efficacy categories, namely 1 student with low self-efficacy, 1 student with moderate self-efficacy, and 1 student with high self-efficacy. The reasons for selecting each student were: 1) the student represented subsequent categories and 2) the student had the ability to communicate and cooperate well to describe their thinking process. The researchers recruited three students as the research subjects which can be seen in Table 2.

Table 2. Research Subjects

Self-Efficacy Category	Student Code
Low Self-Efficacy	TW
Medium Self-Efficacy	SM
High Self Efficacy	CC

Low self-efficacy

TW was a representative of low self efficacy category. The results of TW's answer in determining recursive patterns can be seen in Figure 2. TW made an error in understanding the question, namely, TW answered that the number pattern formed from the number of stickers on each side was 7, 14, 21,…, meanwhile, the correct answer was a pattern of 7, 12, 17,.... However, from the interview results, TW was able to read the questions well.



Translate:

7, 14, 21, ...

Figure 2. Answer TW Number 1 Determine Recursive Patterns

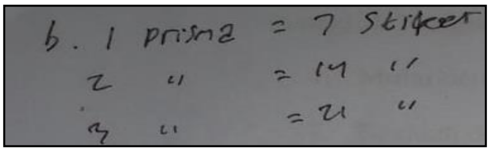
In the next answer, TW determined that the covariance relationship of the pattern that can be seen in Figure 3. In Figure 3, it can be seen that TW had an understanding of the relationship between the number of pentagonal prisms and the number of stickers on each side, namely 1 prism had 7 stickers on each side, 2 prisms had 14 stickers on each side, and 3 prisms had 14 stickers on each side. Prisma had 21 stickers for each side. According to TW, every time a prism was added, 7 stickers were increased. With TW's explanation, he showed that he did not understand the picture in the question well, because the pentagonal prisms were connected to each other, resulting in an increase of 5 stickers instead of 7 for each additional prism sticker. The test results were clarified by the interview’s results that was conducted by researchers with TW. Excerpts from the researcher's interview with TW are as follows:

R : OK, from this picture can you tell the relationship between the flat shape of the prism and the number of stickers on each side?

TW : In the picture of the pentagon prism, there are 7 stickers

R : Is there an increase in the number of building blocks and the number of stickers?

TW : Yes, there is an increase in numbers



Translate:

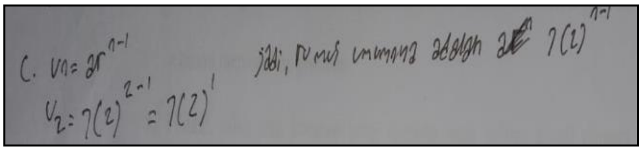
1 Prism = 7 Sticker

2 Prism = 14 Sticker

3 Prism = 21 Sticker

Figure 3. TW Answer Number 2 Determining Covariational Relationships

The results of TW's work in determining correspondence can be seen in Figure 4. Based on Figure 4, it can be seen that TW made errors in understanding, transformation, processing and coding. TW was able to read the questions but was unable to understand the problem, resulting in errors at the covariance stage which resulted in errors in the process of using the formula. TW assumed that problem solving can be done using geometric sequence formulas, but this was not correct because problem solving also used arithmetic sequences. This shows that TW made a mistake in understanding the problem (comprehension error), resulting in a transformation error which showed that TW did not create the correct mathematical model. Not only that, this also resulted in an error in the use of the formula that would be used to solve the problem (process skill error), and incorrectly had a conceptual error (encoding error) which resulted in an error in writing the final answer.



Translate:

$U_n = an^{n-1}$

$U_2 = 7(2)^{2-1} = 7(2)^1$

So the general formula is $7(2)^{n-1}$

Figure 4. TW Answer Number 3 Determining Correspondence

The results of examination of low self-efficacy student based on Newman's error of functional thinking on can be seen in Diagram 1.

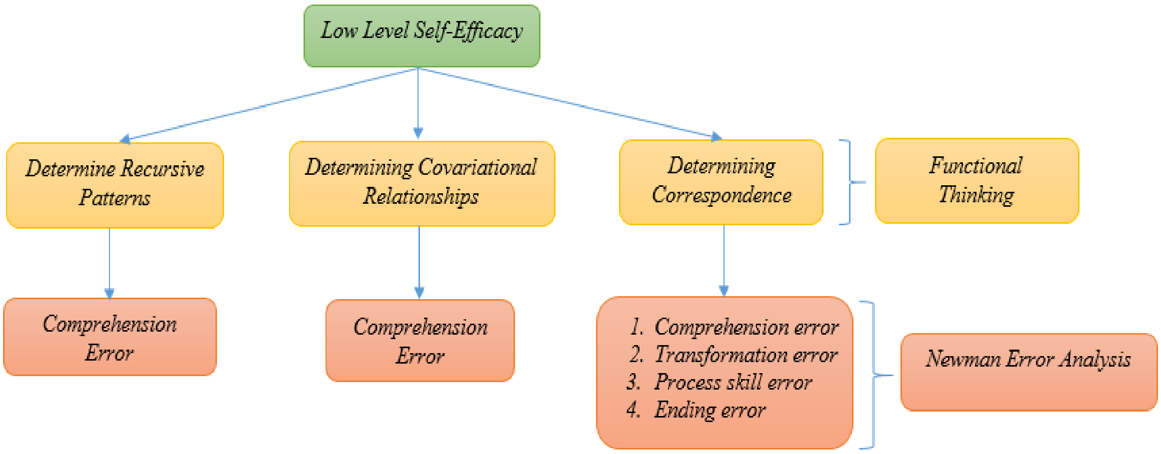
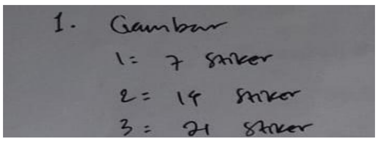


Diagram 1. Newman's Error of Functional Thinking on Low Level Self-efficacy

Medium self-efficacy

SM represented a medium self-efficacy category. SM's answer in determining the recursive pattern can be seen in Table 5. In the answer sheet, it can be seen that SM made an error in understanding the question by assuming that the number pattern formed from the number of stickers on each side was 7, 14, 21,... However, the correct answer should have been 7, 12, 17,... from the interview with SM, it was discovered that SM could read the questions but could not understand the questions well. SM believed that each side of the pentagon prism was all marked with stickers. This showed that SM did not understand the question.



Translate:

Image

1 = 7 Sticker

2 = 14 Sticker

3 = 21 Sticker

Figure 5. SM Answer Number 1 Determining Recursive Patterns

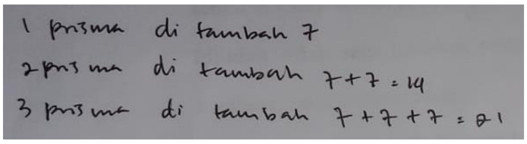
The results of SM's work in determining the covariance relationship can be seen in Figure 6. SM assumed that that each prism increased by 7 stickers. In this case, SM did not understand the picture in the problem well, because in the problem the pentagon prisms were connected to each other which resulted in increasing by 5 stickers for each additional prism sticker. The test results were clarified by interviews conducted by researchers with SM. Excerpts from the interview are:

R : *OK, then can you tell and show the change in value of the relationship between the number of prism shapes and the number of stickers?*

SM : *That's the connection, right ma'am, there are three pictures here, so in this one picture there is a pentagonal prism, then in the second picture there is the addition of another pentagonal prism, likewise in picture three there is one more pentagonal prism added. So, for every addition of a pentagonal prism, there is also an additional sticker, ma'am, so that's the connection*

R : *Does that mean that every time the building space increases, the stickers also increase?*

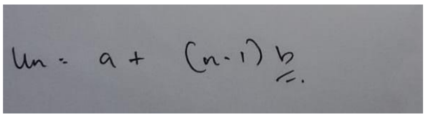
SM : *Yes ma'am*



Translate:
1 Prism plus 7
2 Prism plus 7 + 7 = 14
3 Prism plus 7 + 7 + 7 = 21

Figure 6. SM Answer Number 2 Determining Covariational Relationships

SM completed the questions in determining correspondence presented in Figure 7. SM could read the questions but could not understand the questions, resulting in errors in understanding. SM knows what general formula can be used to find the formula for n pentagonal prisms but does not know how to find the values of *a* and *b*. This results in the SM being unable to continue the problem solving process (process skill error), so that the SM is unable to display the final answer correctly and cannot draw conclusions (encoding error).



Translate: $U_n = a + (n – 1)b$

Figure 7. SM Answer Number 3 Determining Correspondence

The results of examination of a medium self-efficacy student based on Newman's error of functional thinking can be seen in Diagram 2.

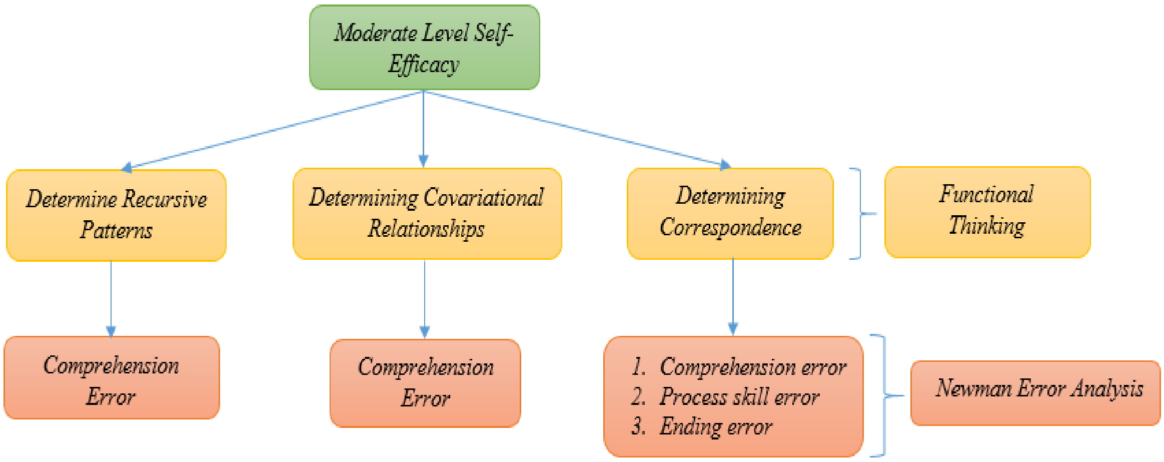
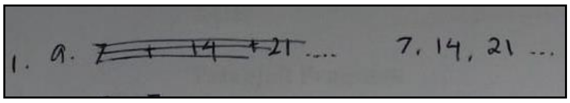


Diagram 2. Newman's Error in Functional Thinking Based on Moderate Level Self-efficacy

High self-efficacy

CC represented a high self efficacy category. CC's answer for a math problem that sought to determine the recursive pattern can be seen in Figure 8. CC also made an error in understanding because according to him the number pattern formed from the number of stickers on each side was 7, 14, 21..., while the correct answer was 7, 12, 17,... However, from the interview, it was revealed that CC could read the questions but could not understand the questions well, whereas CC thought that if the pentagon prism was added then the number of stickers would still count as two stickers. This assumption caused CC to get the number pattern 7, 14, 21,…



Translate: 7, 14, 21, ...

Figure 8. CC Answer Number 1 Determining Recursive Patterns

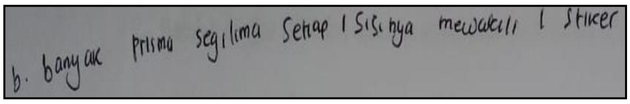
In the next answer CC determined the covariance relationship, which can be seen in Figure 9. CC made an error in understanding because he did not explain in depth how to determine the relationship between the number of pentagonal prisms and the number of stickers. CC's stated that there were many pentagonal prisms with 1 sticker on each side. The explanation given by CC showed that CC did not understand the question. The excerpts from interviews is presented as follows.

R : *Ok, tell me about the change in value of the relationship between the number of pentagonal prisms and the number of stickers on each side?*

CC : *Because the number of pentagonal prisms is different, the stickers also increase, so I just added the number of each sticker.*

R : *If they are side by side, does it still count as two stickers?*

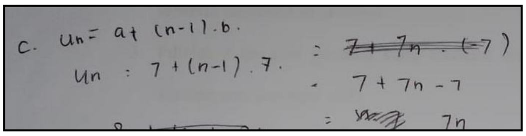
CC : *Yes, I'll still count ma'am*



Translate: b. Many triangular prisms on each side represent 1 sticker

Figure 9. CC Answer Number 2 Determining Covariational Relationships

The next answer was to determine the correspondence relationship, whereas the results of CC's work can be seen in Figure 10. Based on the results of the interview and the results of CC's work, CC was able to read the questions but was unable to understand the problem, which resulted in CC being unable to carry out process skills, resulting in errors in writing the final answer. In CC's answer sheet, it is known that CC could carry out transformations because CC knew the general formula used to find the number of stickers on n pentagonal prisms. From the interview CC said that the general formula was an arithmetic sequence. Therefore, CC could have carried out the transformation because CC already knew what general formula can be used to find out the number of stickers if n pentagonal prisms, but unfortunately because he misunderstands the problem, CC could not continue the solving process to find the correct answer.



Translate:
$U_n = a + (n-1)b$
$U_n = 7 + (n-1)7 = 7 + 7n-7 = 7n$

Figure 10. CC Answer Number 3 Determining Correspondence

The results of examination of a high self-efficacy student based on Newman's error of functional thinking can be seen in Diagram 3.

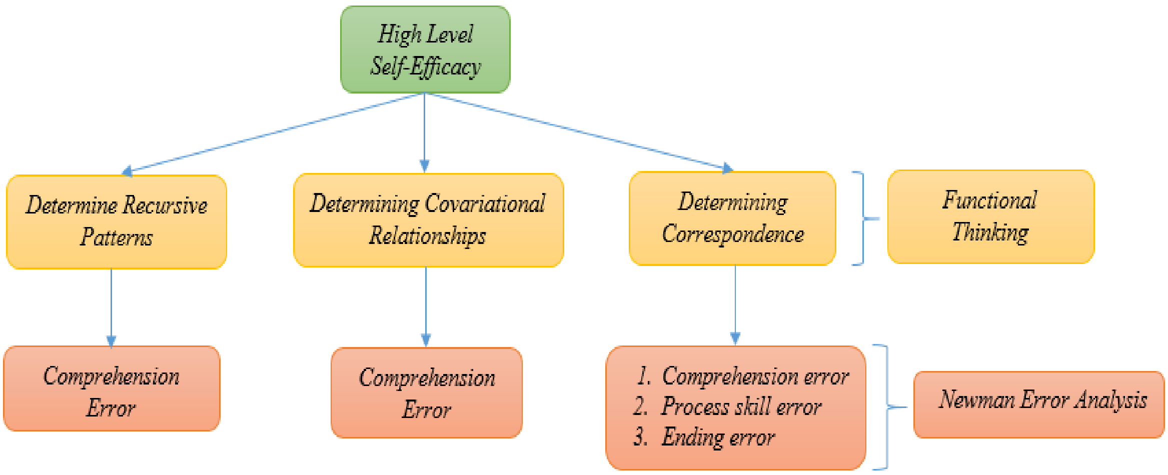


Diagram 3. Newman's Error in Functional Thinking Based on High Level Self-efficacy

Based on the discussion above, at the stage of determining the recursive pattern and covariational relationships, low, medium, and high self-efficacy students made the same mistakes, namely errors in understanding. Errors in understanding consist of students' errors in not writing what is known and asked or writing what is known and asked but incorrectly. This is in line with research by (Fatahillah et al., 2017) (Mahmudah, 2018) which found that the largest percentage of errors was in errors in understanding because most students were not able to understand the questions correctly, there were still many errors in understanding the commands and things asked in the questions. In determining correspondence, low self-efficacy students made errors in understanding, transformation errors, process skill errors, and answering errors (enconding errors). This is in line with research conducted by (Murtiyasa & Wulandari, 2020) (Jami et al., 2020) who found the same mistakes in solving story problems. Meanwhile, students with medium and high self-efficacy made errors in understanding, process skill errors and final answer errors (enconding errors). The results of the study Amalia (2017) also found the same mistakes in solving math problems. Many factors cause students to make mistakes in solving problems, namely students are still confused in understanding the problem, are not careful when working on the problem, do not understand the symbols and do not have time to write conclusions.

Thus, there is a relationship between self-efficacy and student errors in functional thinking. Students with high and medium self-efficacy tend to make fewer functional thinking errors than students with low self-efficacy. Students with low self-efficacy make more mistakes in solving functional thinking problems. Students' functional thinking abilities can be tracked through the problem solving they do. Problem solving carried out by students is not only about the final answer, but the solving process itself must also be considered. In solving math problems, students are expected to follow a step-by-step process so that they can track their own thought process (Warren et al., 2013). According to Wilkie (2014) the process of developing students' functional thinking can gradually start with simple patterns and continue to more complex patterns. While self-efficacy is a person's belief in their ability to achieve goals and predict how much effort is needed to achieve those goals. Students with high self-efficacy in learning or doing assignments tend to participate further, work harder, and persist longer even when facing difficulties and as a result they will achieve higher levels of achievement. In other words, the learning outcomes of students with high self-efficacy are certainly higher than those with low self-efficacy (Yoannita et al., 2016; Yoni, 2017; Döş, 2023).. Therefore, self-efficacy is relevant to students' ability to solve mathematics problems and the types of errors they make. Previous studies have also used Newman's errors to investigate student errors and reported that there are five types of errors (Ningsih et al., 2021) and (Arumiseh et al., 2019) (Kurniati et al., 2021) (Jami et al., 2020). The findings of this study are presented in table 3.

Table 3. Research Findings

Self Efficacy Questionnaire	Functional Thinking Framework	Newman Error Committed	Information
Low self-efficacy students	Define a recursive pattern	Comprehension error	Students do not understand the instructions on the questions which results in errors in designing solutions so they cannot solve the questions correctly, and errors in writing the final answer. This is supported by
	Determining covariational relationships	Comprehension error	
	Determining correspondence	Comprehension error Transformation error	

Self Efficacy Questionnaire	Functional Thinking Framework	Newman Error Committed	Information
		Process skill error Ending error	interviews conducted that students make mistakes in using general formulas.
Medium self-efficacy students	Define a recursive pattern Determining covariational relationships Determining correspondence	Comprehension error Comprehension error Comprehension error Process skill error Ending error	Students do not understand the commands in the questions which results in designing solutions incorrectly so they cannot solve the questions correctly. Students already know what formula to use, but students are confused about how to solve it, which results in students not being able to carry out the solving process properly, and not being able to write the final answer.
High self-efficacy students	Define a recursive pattern Determining covariational relationships Determining correspondence	Comprehension error Comprehension error Comprehension error Process skill error Ending error	Students do not understand the commands in the questions which results in designing solutions incorrectly so they cannot solve the questions correctly. Even so, students already know what formula to use, but because they misunderstand the question, it results in errors in writing the final answer.

CONCLUSION

Low self-efficacy students faced significant difficulties across all stages, indicating a fundamental struggle in understanding and solving functional thinking problems. Moderate self-efficacy students exhibited better conceptual understanding, but their errors in process skills and final answer writing suggest a need for targeted instructional support. High self-efficacy students demonstrated stronger problem-solving abilities, with errors primarily related to procedural execution rather than conceptual understanding. These findings reinforce the importance of self-efficacy in mathematical problem-solving and suggest that students with higher self-efficacy are more persistent and capable of progressing through complex mathematical reasoning despite occasional errors. Future instructional strategies should focus on strengthening comprehension skills and procedural accuracy to minimize functional thinking errors at all self-efficacy levels.

Acknowledgment

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