

Junior High School Students' Proportional Reasoning Ability in the Domain Comparison: A Gap in Results Between Indicators

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Abstract

The purpose of this study was to analyze students' proportional reasoning ability. The research subjects consisted of 21 students of VIII class in one of the public junior high schools in Semarang City who had obtained comparison materials (ratio and proportion). Data collection used test instruments that were developed based on indicators of proportional reasoning ability consisting of 6 questions. The data were analyzed qualitatively with data reduction analysis procedures, data presentation, and conclusion drawing/verification. The results showed that (1) none of the students met all the indicators of proportional reasoning ability, namely the ability to understand proportional relationships, solve proportional problems of missing values, numerical comparisons, qualitative predictions, distinguish proportional and non-proportional problems, and solve proportional relationships of inverse values, (2) all students have been able to understand proportional relationships, but they are still very limited in their ability to distinguish proportional and non-proportional problems, and solve proportional relationships of inverse value, and (3) the strategy most often used by students to solve proportional reasoning problems is the cross-multiplication strategy.

Keywords: comparison, junior high school, proportional reasoning, proportion, ratio

INTRODUCTION

Reasoning ability is one of the central concepts in mathematics education (Kollosche, 2021), both in research and classroom practice (Hjelte et al., 2020). It is also stated in the main objectives of the mathematics education curriculum (Arshad et al., 2017; Jeannotte & Kieran, 2017; Smit et al., 2022). This concept is vital in mathematics education because mathematical reasoning ability makes an important contribution to students' overall critical thinking skills and it enable students to use their logic in facing challenges both in the classroom and in everyday life (Mukuka et al., 2023). Students who have good reasoning skills can lead them to better understand and apply



mathematical concepts in daily life (Marasabessy, 2021). It not only prepares them for exams or academic tasks, but also equips them with the skills needed in the real world where logical thinking and reasoning ability are key to success.

Reasoning can be defined as a line of thought and a way of thinking used to generate assertions and draw conclusions (Lithner, 2000). Simply, reasoning can be defined as the process of finding reasons or causes to conclude or draw conclusions (Khan & Ullah, 2010). In addition, reasoning can also be defined as the flow of thinking in generating statements and reaching conclusions in solving tasks or problems (Sidenvall et al., 2015). In making conclusions, students can use appropriate logic, evidence, and arguments (Walton, 1990). Based on some explanations, it can be concluded that the reasoning process uses logic, evidence, and arguments to draw conclusions, make decisions, or solve problems.

Mathematical reasoning is the ability to use logic and critical thinking to analyze and justify mathematical statements, arguments, and proofs. This is in line with the statement of Hasanah et al. (2019) which states that mathematical reasoning is a process carried out to obtain conclusions based on mathematical logical premises and facts that have been proven. Mathematical reasoning is needed to determine whether a mathematical argument is true or false and it is used to build an argument (Agustin, 2016). Mathematical reasoning ability allows students to be able to solve mathematical problems well and allows students to solve their daily life problems.

There are several reasoning abilities that are developed in learning mathematics, one of which is proportional reasoning ability (Geçici, & Türnüklü, 2021). It is a basic aspect of school mathematics (Weiland et al., 2021). Proportional reasoning ability is an important concept from elementary to college level mathematics, it also has an important role and has several practical functions (Ekawati et al., 2015), both in mathematics, other subjects, and daily life (Ayuningtyas, 2019). Students who have good proportional reasoning skills lead them to understand quantity relationships in a comparison (Lutfi et al., 2021). It makes proportional reasoning become one of the most intensively studied topics in mathematics education research (Wijayanti & Winslow, 2017).

Each type of reasoning ability emphasizes certain aspects. Proportional reasoning, for example, emphasizes the thinking process involving ratio and proportion (Livy & Herbert, 2013). In other words, proportional reasoning is the ability to reason related to proportional situations (i.e. related to ratio and proportion). Ratio is a situation in terms of comparison (Dole et al., 2012), in which it can be written in the form of $\frac{a}{b}$ or $a : b$ that expresses a multiplicative relationship between two quantities (Keersmaeker et al., 2023). While, proportion is a statement of equality of two ratios that can be expressed in $\frac{a}{b} = \frac{c}{d}$ form (Tourniaire & Pulos, 1985; Prayitno et al., 2018).

Proportional reasoning ability can also be interpreted as the ability in mathematics on multiplicative relationships (Muttaqin et al., 2017). Multiplicative relationships or proportionality relationships can be understood through scalar relationships and functional relationships (Carney et al., 2022; Hino & Kato, 2019). Scalar relationships describe the scale factor by which each quantity in a ratio can be multiplied or divided to produce an equivalent ratio. While, functional relationships describe the constant multiplication factor that exists between two quantities in a rate situation. Proportional

reasoning ability is the ability of students to solve proportionality problems or multiplicative relationship problems such that if one quantity changes, then the other also changes. Proportional reasoning ability is found in several materials including scale, probability, percentage, rate, trigonometry, equality, measurement, algebraic geometry (Misnasanti et al., 2017), and comparison (Puspita et al., 2023). From some of these materials, proportional reasoning ability will be much needed in comparison or ratio materials.

In mathematics learning, students often have difficulties in solving proportional reasoning problems. The difficulties faced by students are in finding multiplicative relationships, understanding inverse value comparisons, and explaining the results obtained (Mardika & Mahmudi, 2021), distinguishing proportional from non-proportional situations (Tjoe & Torre, 2014). Frequent errors in solving proportional reasoning problems are caused by confusion in identifying unit values and algorithm-based errors in the computational process (Soyak & Isiksal, 2017).

Based on the previous explanation, it is known that proportional reasoning has a fundamental role in mathematics learning. Therefore, research on proportional reasoning needs to be conducted, one of which is to find out how students' proportional reasoning skills. Similar research conducted by Prayitno et al. (2018) only used proportional reasoning questions of missing value problem type on direct proportion, the results showed that the proportional reasoning ability of VIII grade students has not reached the optimal stage. Further research conducted by Mardika & Mahmudi (2021) used three proportional reasoning problems, namely 1) missing value problem on direct proportion; 2) missing value problem on indirect proportion; and 3) comparison problem. In addition, Zulkarnaen (2017) also conducted a study aimed to analyze the proportional reasoning ability of grade X students using three problems, namely 1) missing value problem; 2) numerical comparison; and 3) qualitative prediction; by classifying students based on high, medium, and low school categories. Based on the literature, the novelty of this study lies in the use of six types of problems in proportional reasoning completely, namely 1) proportionality relationship; 2) missing value problem in direct proportion; 3) numerical comparison problem; 4) qualitative prediction problem; 5) distinguishing proportional problem and non-proportional problem; and 6) missing value problem in indirect proportion.

Therefore, this study aims to analyze the proportional reasoning ability of VIII grade students on comparison material. In contrast to previous studies that have not used the six types of proportional reasoning problems, this study has a novelty in the form of proportional reasoning test instruments used. The use of the six types of proportional reasoning problems will certainly provide a comprehensive overview of how students' proportional reasoning ability.

METHODS

This study aims to examine the proportional reasoning ability of junior high school students on comparative material, so researchers use a qualitative approach. It is because the qualitative approach will emphasize the exploration and meaning of a problem

(Creswell & Creswell, 2022). Descriptive method was chosen in analyzing students' proportional reasoning ability which positioned the researcher close to the data by using framework and interpretation to explain the data (Creswell & Creswell, 2022). This study was conducted at a public junior high school in Semarang City in the 2023/2024 academic year. The subjects consisted of 21 students of VIII grade who had learned the comparison (ratio and proportion) material in the previous grade. Students were given a proportional reasoning ability test and directed to write their answers according to their abilities in detail. The subjects have also agreed that they are involved in the research.

Research procedure

Since the research is proceeding through a series of different steps, it utilizes the procedure presented in Figure 1 as follows (Creswell, 2012):



Figure 1. Research Procedure

Based on Figure 1, this study began with identifying research problems, this study was motivated by the importance of proportional reasoning ability as an ability that becomes a factor of success in advanced mathematics, so this research will provide an overview of how the proportional reasoning ability of students at the junior high school level. At the stage of reviewing the literature, the researchers examined proportional reasoning which includes definitions and indicators, also the extent to which students' proportional reasoning abilities that have been studied before. In the process of preparing the proportional reasoning ability instrument, the researchers constructed a specification table which was arranged based on the material, indicators of proportional reasoning ability, question indicators, question numbers, questions, alternative answers, and score weights. The instrument that has been prepared by the researchers is then presented to three experts in the field of mathematics education for the purpose of question suitability before being used as an instrument. Revisions were made based on expert suggestions that could refer to the grade level and related objectives. At the stage of determining research objectives, the researchers determined the focus of the research, namely to identify proportional reasoning abilities which will be presented in the order of general classical achievements, achievements of each indicator, strategies used, and difficulties faced by students. The researchers collected data through proportional reasoning ability test and documentation of test results. At the stage of analyzing and interpreting data, this procedure will be explained more fully in the analysis section. Finally, the researchers made a report that contained research conclusions and evaluating the research.

Proportional reasoning ability instrument

To measure students' proportional reasoning ability, it required a series of tasks that are organized based on proportional reasoning indicators. In this study, the question instrument was developed based on indicators: 1) the ability to understand mathematical relationships contained in proportional problems or multiplicative relationships; 2) the ability to solve missing value problems in direct proportion; 3) the ability to solve numerical comparison problems; 4) the ability to solve qualitative prediction problems;

5) the ability to distinguish proportional problems and non-proportional problems; and 6) the ability to solve missing value problems in inverse proportion (Ayuningtyas, 2019; Cramer et al., 1993). The proportional reasoning instrument used in this study was presented in Figure 2:

1. The ratio of Adit and Budi's current age is 3: 4. If the sum of Adit and Budi's ages is 28 years. What is the difference between Adit and Budi's ages? Include your steps in a coherent and clear manner!
2. A grinding machine requires $3\frac{1}{2}$ liters of gasoline and $\frac{1}{2}$ liters diesel to produce 2 quintals of corn in 2 hours. If the corn available is 1 ton. The length of time and the amount of gasoline and diesel needed are? Write down your working steps!
3. There are two types of chocolate boxes to choose from. Box I contain 4 boxes of chocolate with 21 pieces of chocolate in each box. Box II contains 3 boxes of chocolate with 28 pieces of chocolate in each box. If Box I is sold at Rp45,000 and Box II is sold at Rp60,000. Which chocolate box option is cheaper? Why? Include your working steps!
4. If Dodi mixes less fruit syrup with more water than he did yesterday. Will the drink taste stronger or weaker or exactly the same? Or is there missing information? Provide a clear argument to build your answer choice!
5. A proportion $\frac{1}{2} = \frac{10}{x}$ is known. There are two situations as follows:
Situation I: For every 1 boy there are 2 girls in a class. If there are 10 boys in the class, how many girls are there?
Situation II: Bob is 1 year old and Mary is 2 years old. When Bob is 10 years old, how old is Mary?
 Of the two situations above, which one of them is solved according to the given proportion $\frac{1}{2} = \frac{10}{x}$? Answer option:
 - a. Situation I only
 - b. Situation II only
 - c. Situation I and II
 - d. Situations I and II do not fit the given proportion
 Include clear reasons and steps to solve the problem.
6. A road repair project was completed in 30 days with 15 workers, after 6 days of implementation, for some reason the project was stopped for 4 days. If the ability of each worker is the same and for the project to be completed on time, the additional workers needed are? Explain your answer clearly!

Figure 2. Proportional Reasoning Ability Instrument

The six problems in Figure 2 were used to measure the complete proportional reasoning ability. In contrast to Zulkarnaen (2017) who used missing value problem, numerical comparison, and qualitative prediction, Yuliani et al. (2021) who used proportional relationship and value and inverse value comparison, and Ayan & Isiksal-Bostan (2019) who used value comparison and inverse comparison problems, this study used the six problems on adaptation of some of these studies, because the researchers considered it important to analyze students' proportional reasoning ability based on all components in proportional reasoning.

Analysis

The general analysis of proportional reasoning ability was based on the categories in Table 1 (Suprihatin et al., 2018), and the indicators used to analyze students' proportional reasoning ability were 1) expressing proportional relationships into mathematical models; 2) analyzing proportional relationships; 3) explaining the answers given regarding proportional relationships (Lamon, 2012).

Table 1. Category of Proportional Reasoning Ability

No	Description	Category
1	Percentage > 70%	High
2	$50\% \leq \text{Percentage} \leq 70\%$	Medium
3	Percentage < 50%	Low

To analyze the data, the procedures of data reduction, data presentation, and conclusion drawing/verification were used. Data reduction was done by summarizing the characteristics of student answers, and classifying the main and important student answers to facilitate researchers in data preparation. Data presentation can be done in the form of

several different student answers, all student answers that have differences are presented in this study to provide an overview of student completion on each problem. Drawing conclusions was based on strong, valid, and consistent evidence to produce valid and credible conclusions.

RESULTS AND DISCUSSION

This section presents the general achievement and description of proportional reasoning ability that has been obtained based on data collection on VIII grade junior high school students on comparison materials (ratio and proportion). The results of this study are described based on the sequence of questions used to analyze students' proportional reasoning ability.

General achievement of proportional reasoning ability

The students' answers were assessed to find out the general overview of the overall achievement of students' proportional reasoning ability. Table 2 presents the results of the overall achievement of students' proportional reasoning abilities that were categorized based on the three categories described in the method section of this study.

Table 2. General Achievement of Proportional Reasoning Ability

No	Category	Percentage
1	High	0,00%
2	Medium	19,04%
3	Low	80,96%

Based on Table 2, there are 80,96% of students who have low proportional reasoning ability, 19,04% of students have medium proportional reasoning ability, and there are still no students who have proportional reasoning ability in the high category. Based on the scoring that has been done, the average score is 35.31% or in the low category. This is in line with Yuliani et al. (2021) that most students still have low proportional reasoning skills. From the overall student answers, it shows that students have been able to know the multiplicative relationship in the given proportional situation.

Based on the six indicators used, students are most successful in solving problems in the first indicator, the ability to understand mathematical relationships contained in proportional relationships. Meanwhile, the missing value problem in indirect proportion is a problem that no student has been able to solve well. This is in line with research Arican (2019) that students are more successful in solving direct proportional relationships than indirect proportional relationships.

The strategy most often used by students is appropriate, namely cross multiplication, this is in line with research Özgün-Koca & Altay (2009) and Ayan & Isiksal-Bostan (2019) that students more often use the cross multiplication strategy. This strategy based on Avcu & Doğan (2014) is said to have no physical referent and less meaningful to students, and it is recommended not to be introduced to students first before students gain knowledge of conceptual-based strategies, such as factor of change strategy, build-up strategy, or unit rate strategy.

In the next section, students' answers to each problem are presented based on the information that can be presented to provide sufficient information. The selected answers have been grouped based on the information that is covered from all student solutions. Due to the limited strategies used by students, the answers that were presented will also be limited because other answers will not provide additional information.

Students' answers to the first problem

The indicator of the first problem is the ability to understand mathematical relationships contained in proportional relationships in the form of $\frac{a}{b} = \frac{c}{d}$ (Thurn et al., 2022). All students (100%) have shown the correct final answer. The context of the first problem is about the ratio and the sum of the ages of the two children that is known. The students' task is to determine the age difference of the two children. Figure 3 presents the students' answers in solving the first proportional reasoning problem:

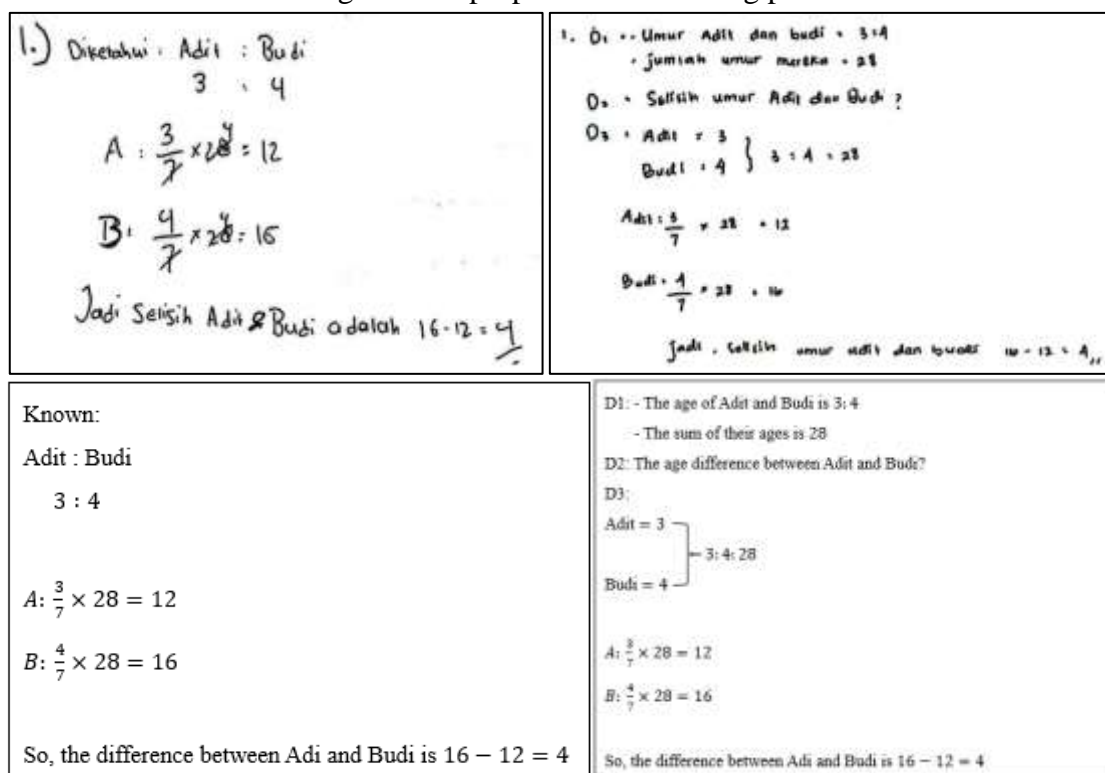


Figure 3. Students' answers to the first problem

The two answers in Figure 3 were selected on the basis of the overall representativeness of students' answers to the first question. All students (100%) used the same strategy in determining the difference between the ages of two children, namely by determining the age of each child first, then proceeding with the subtraction operation to determine the difference between the ages of the two children. Based on these two answers, it shows that students have been able to determine a mathematical model based on the given multiplicative relationship. It can be seen in stating the proportional relationship to determine the ages of Adit and Budi based on the ratio given, but in terms of stating the information contained in the problem, there are still many students (80.95%) who are not correct in stating the overall ratio relationship formed from the problem. Students only stated the ratio between the ages of Adit and Budi, without paying attention

to the sum of their ages. In addition to stating the ratio of Adit and Budi's ages, namely 3: 4, students also state the ratio of the number of ages formed from the ratio $A + B = 7$, which refers to the quantity of the number of ages, namely 28 years. It shows that they still have a limited understanding of fraction ratio representation (Im & Jitendra, 2020).

Students' answers to the second problem

The indicator of the second problem is the ability to solve proportional problems of the missing value problem in direct proportion. It is a proportional reasoning problem that contains three known quantities with one quantity to be sought (Tourniaire & Pulos, 1985; Arican, 2019; Son, 2013). The second problem context is about a proportional situation related to the relationship between gasoline, diesel fuel, and the time that is needed to grind a certain amount of corn, with a known rate. The student's task is to find the unknown value of time, gasoline, and diesel fuel needed. All students (100%) showed the same final answer, which was correct in determining the amount of time needed but still had errors in determining the amount of gasoline and diesel needed. In Figure 4, the students' answers in solving the second proportional reasoning problem are presented.

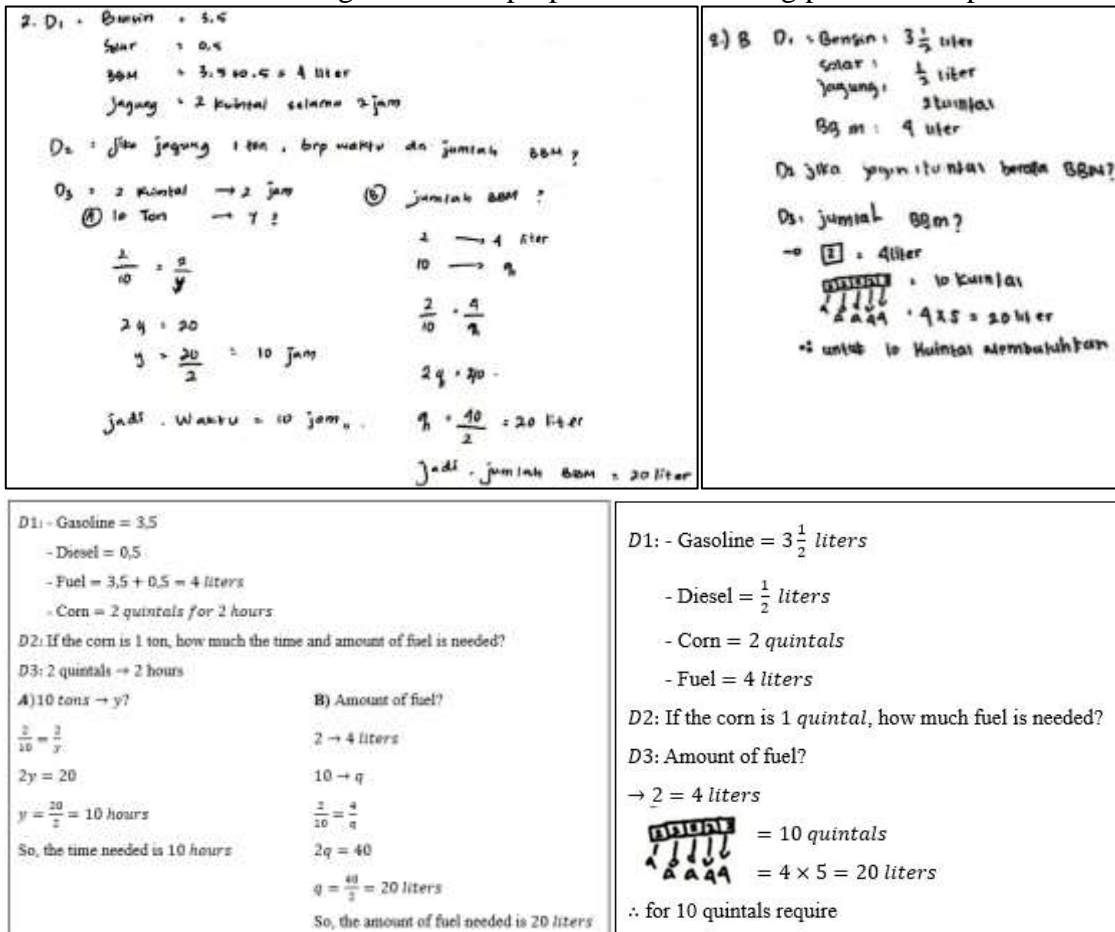


Figure 4. Students' answers to the second problem

Based on Figure 4 on the left, it can be seen that students have been able to form a functional relationship (Carney et al., 2022; Hino & Kato, 2019), namely the relationship regarding every 2 quintals of corn takes 2 hours to grind the corn, even though in solving it students still use the cross times strategy (Arican, 2019). This reinforces the results of

research which states that students more often use the times cross strategy, a traditional strategy that is often taught by teachers in class (Ayan & Isiksal-Bostan, 2019; Özgün-Koca & Altay, 2009). In fact, this strategy is loaded with meaning or conceptually weak, so it is not uncommon for teachers to avoid using this strategy in class (Arican, 2018). The second problem was to determine the need for gasoline and diesel to grind 1 ton or 10 quintals of corn. Based on Figure 4 on the right, students stated gasoline and diesel into a unit, namely BBM, so that students made mistakes in answering the question and it was done by all students (100%) in this study. This finding supports the statement that students often face errors in solving proportional problems that contain fractional numbers compared to ratios that only contain whole numbers (Kaput & West, 1994). In Figure 4 on the right, it can be seen that students used the building up strategy (Koellner-Clark & Lesh, 2003), a strategy to determine the amount of fuel (BBM) needed to grind a certain amount of corn.

Students' answers to the third problem

The indicator of the third problem is the ability to solve numerical comparison problem, a problem with the form of the four quantities that make up the proportion given, the aim of the problem is to determine whether the two are the same or whether one ratio is higher or smaller than the other ratio (Ben-Chaim et al., 2012; Son, 2013). The context of the third problem is about comparing the price of a chocolate with different packaging. Students are allowed to determine the cheaper chocolate option from the two available options. In Figure 5, students' answers in solving the third proportional reasoning problem are presented.

Figure 5 above shows that 19 students (90.47%) have been able to solve numerical comparison type of the proportional reasoning problems, although there is one student (4.76%) who does not state the comparison relationship between the two ratios and one student (4.76%) who has not been able to state the steps and solutions. However, in stating the reasons regarding the numerical comparison relationship of the two quantities given, there are still 13 students (61.90%) who have not been able to state the reasons exactly why chocolate in the Box I is cheaper, and there are only 6 students (28.57%) who state the logical reasons for decision making regarding the given ratio relationship. This shows that students are still not good enough in mastering numerical comparison type problems. In Figure 5 on the left shows that students have been able to show that Box I is cheaper, but the student's reasoning is still inaccurate. While in Figure 5 on the right there are two answers, students have been able to formulate the right reason why Box I is cheaper, namely because Box I and Box II have the same amount of chocolate content with a smaller price for Box I. This shows that students are still not good enough in mastering numerical comparison type questions.

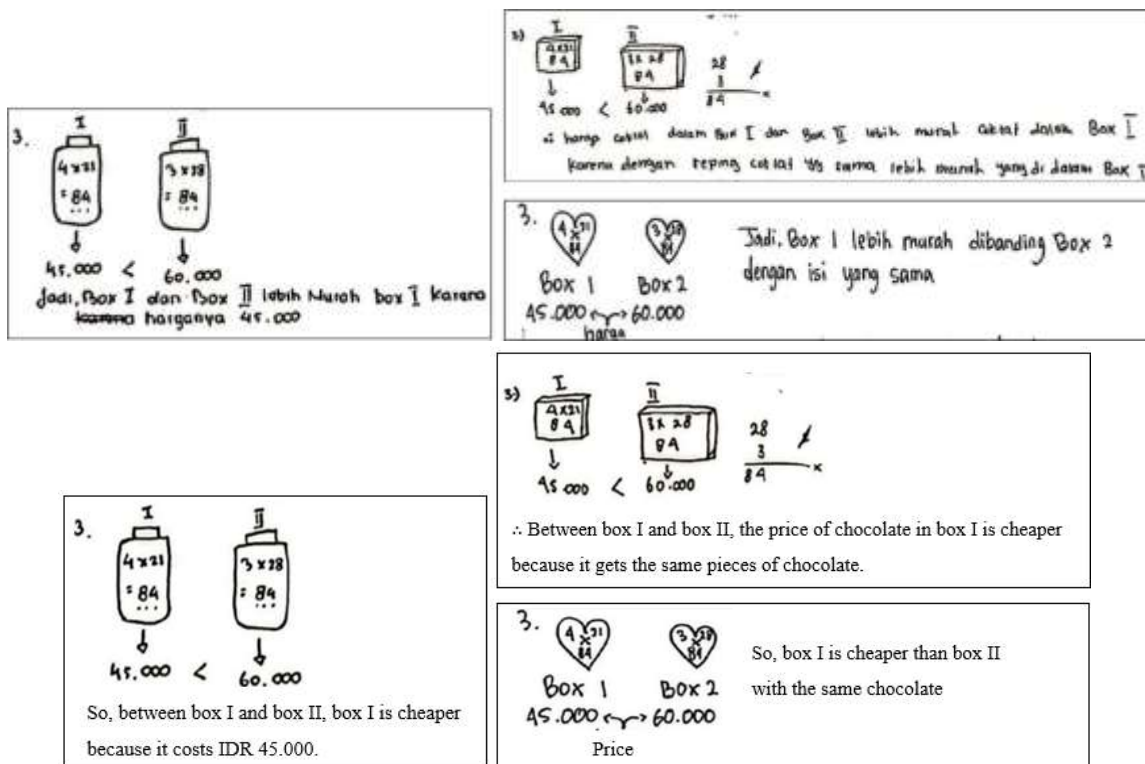


Figure 5. Students' answers to the third problem

Students' answers to the fourth problem

The indicator of the fourth problem is the ability to solve proportional problems of the qualitative prediction type. In this problem, there is no specific numerical value, the aim is to make a comparison between two ratios regardless of numerical value (Cramer et al., 1993; Son, 2013). The context of the fourth problem is about making a drink by mixing syrup and water. Students are allowed to determine the taste that will occur when compared to the previous mixture. Figure 6 presents students' answers to the fourth proportional reasoning problem.

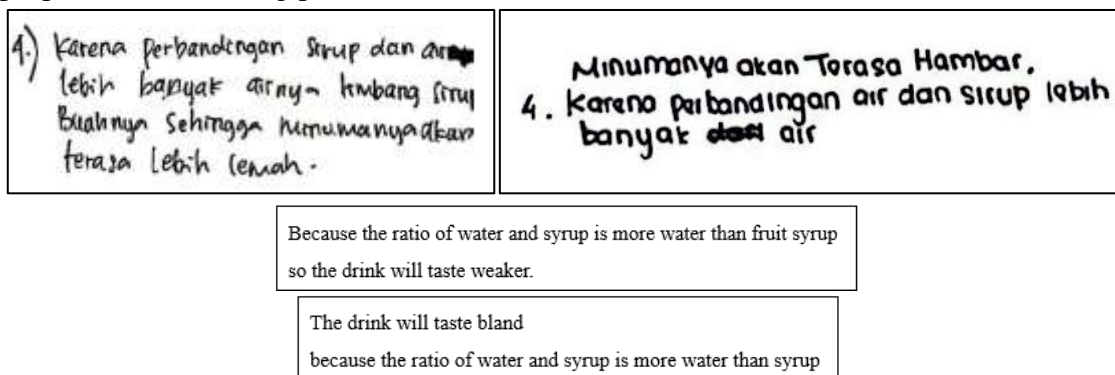


Figure 6. Students' answers to the fourth question

In the fourth question, there were three kinds of student answers. Based on Figure 6 on the left, students could state that the taste of the drink would be weaker than the previous drink because the ratio of water and syrup was more syrup. This answer was not correct because what made the taste of the drink weaker actually was the quantity of water

that was increased and the quantity of syrup that was reduced. There were 3 students (14.28%) who have similar answer, but this answer was much better than the other answers. 13 students (61.90%) stated that the drink made would be tasteless or bland, this does not answer the problem at all because what is asked in the problem is to determine whether the taste of the drink will be the same, stronger, or even weaker. In addition, there were also 5 students (23.80%) who answered this qualitative prediction problem unrelated and incomplete. This finding is contrary to research (Öztürk et al., 2021) which states that students are more successful in solving numerical comparison problems than qualitative prediction.

Students' answers to the fifth problem

The indicator of the fifth problem is the ability to distinguish proportional problems and non-proportional problems. This problem allows students to be able to distinguish problems that use multiplicative relationships or use additive relationships. In Figure 7 below, students' answers in solving the fifth problem are presented.

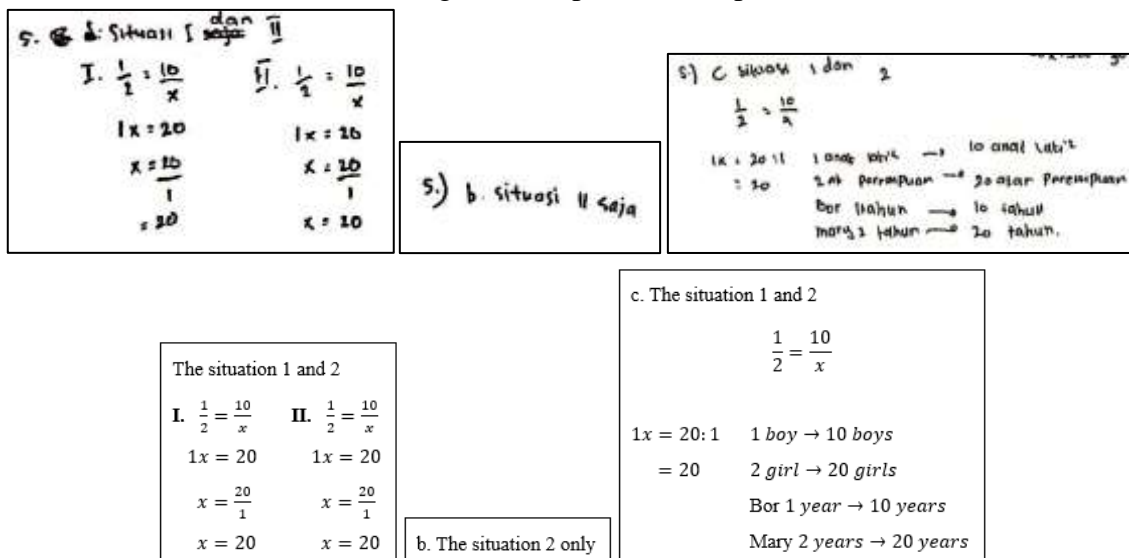


Figure 7. Students' answers to the fifth question

Based on the answers analyzed, all students (100%) have not been able to distinguish proportional and non-proportional relationships. This finding is in line with research (Avcu & Doğan, 2014) that students have difficulty in distinguishing proportional relationships and non-proportional relationships. It can be seen from 19 students (90.47%) who stated that Situation I and Situation II in the problem can be solved with the given proportions presented in Figure 7 left and right parts, 1 student (4.76%) stated that Situation II only is in accordance with the given proportions presented in Figure 7 middle part, and 1 student (4.76%) did not provide any answer information. This fifth question allows students to differentiate between proportional relationships that can be expressed in multiplicative comparison relationships and non-proportional relationships that fulfill additive comparison relationships, which should only be Situation I that fulfills the given proportion. Majority of students' answers stated that the proportions in Situation I and Situation II were continued by operating them using cross-validation, even though the proportions formed did not match the given proportions. It

indicated that students still used the cross-validation strategy on incompatible proportion relationships (Nasir, 2018).

Students' answers to the sixth problem

The indicator of the sixth problem is the ability to solve missing value problems in indirect proportion problems, a proportion that has the form $xy = k$, where k is a constant of proportionality (Andrusiak, 2007). In Figure 8 below, students' answers in solving the sixth problem are presented.

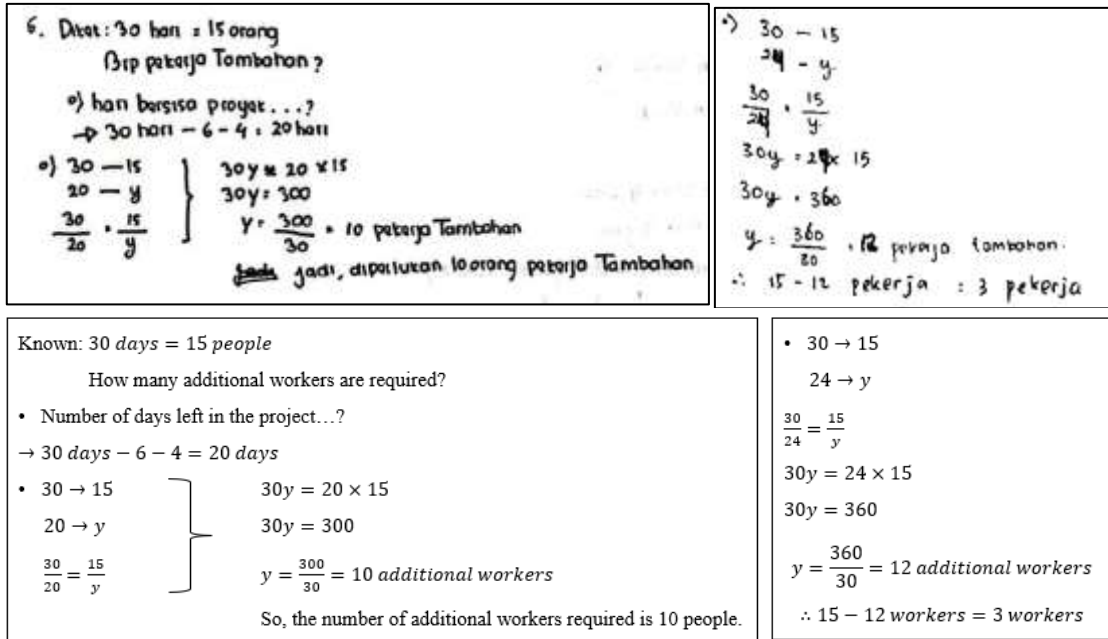


Figure 8. Students' answers to the sixth question

In the sixth question, there were 19 students (90.47%) who answered using the proportional relationship presented in Figure 8 on the left and 2 students (9.53%) answered using the proportional relationship in Figure 8 on the right. However, based on the two forms of proportion made by students, there is no appropriate proportion relationship from the question. The two proportions stated in the two figures above are formed from direct proportion relationships, so they are not in accordance with the context of the problem given which should use indirect proportion relationships. The finding of research (Mardika & Mahmudi, 2021) also showed the same thing that students are more proficient in solving direct proportion problems than indirect proportion. The unique in this study is that there are no students who can state the indirect proportion relationship given in the problem. In addition, the proportion formed by the students in the answer is a proportional value, indicating that students are still mastering the inverse proportion relationship because the missing value problem in indirect proportion is the most difficult problem for students to solve compared to other types of problems in proportional reasoning (Avcu & Doğan, 2014).

CONCLUSION

Based on the results and discussion that have been presented, it can be concluded that the proportional reasoning ability of class VIII students on comparison (ratio and proportion) material is not optimal as a whole. The results of the overall analysis of students' answers showed that the most successful students answered the first indicator, namely the ability to understand mathematical relationships in proportional relationships with 100% students could answer the problem, indicating that it is a satisfactory result. In the second indicator, the ability to solve proportional problems of missing value problems in direct proportion, there were 50% of students who answered correctly, 90.47% of students successfully answered the third indicator, the ability to solve numerical comparison type problems. In the fourth indicator, the ability to solve proportional problems of qualitative prediction type, only 14.28% of students have answered correctly. Meanwhile, In the fifth indicator, the ability to distinguish proportional problems and non-proportional problems, and the sixth indicator, the ability to solve inverse comparison problems, there are no students who have mastered it. Further research that can be conducted is about solutions to improve students' proportional reasoning by applying certain learning models, approaches, methods, and strategies, or by using media integrated with technology.

This research provides a complete description of how the proportional reasoning ability of class VIII students through six indicators. This finding implies that it is necessary to facilitate students in differentiating proportional and non-proportional problems and the problem of inverse comparison, because there are still no students who master it well to prepare students for better proportional reasoning skills. In addition, students' mastery of problems involving qualitative prediction is also still limited. This clearly provides room for contribution to future learning implementation.

The limitations of this study regarded proportional reasoning indicators that can differ from one researcher to another. However, researchers have tried to use indicators to measure students' proportional reasoning abilities well by adopting several related studies. Because this study used a relatively small sample, the next research can use a larger sample size so it can provide more complete information. This study also has not examined how students' reasoning abilities between different classes, for example from grades six, seven and eight can be compared, and student gender also needs to be considered for future research.

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